MUSIC-Based Pilot Decontamination and Channel Estimation in Multiuser Massive MIMO System
Wei-Chiang Wu

Citation: Wu Wei-Chiang. MUSIC-Based Pilot Decontamination and Channel Estimation in Multiuser Massive MIMO System[J]. Journal of Electronic Science and Technology, 2020, 18(3): 266-275. doi: 10.11989/JEST.1674-862X.80102112

View online: https://doi.org/10.11989/JEST.1674-862X.80102112

Articles you may be interested in

Follow JEST WeChat public account for more information
MUSIC-Based Pilot Decontamination and Channel Estimation in Multiuser Massive MIMO System

Wei-Chiang Wu*

Abstract—This paper addresses the problem of channel estimation in a multiuser multi-cell wireless communications system in which the base station (BS) is equipped with a very large number of antennas (also referred to as “massive multiple-input multiple-output (MIMO)”). We consider a time-division duplexing (TDD) scheme, in which reciprocity between the uplink and downlink channels can be assumed. Channel estimation is essential for downlink beamforming in massive MIMO, nevertheless, the pilot contamination effect hinders accurate channel estimation, which leads to overall performance degradation. Benefitted from the asymptotic orthogonality between signal and interference subspaces for non-overlapping angle-of-arrivals (AOAs) in the large-scale antenna system, we propose a multiple signals classification (MUSIC) based channel estimation algorithm during the uplink transmission. Analytical and numerical results verify complete pilot decontamination and the effectiveness of the proposed channel estimation algorithm in the multiuser multi-cell massive MIMO system.

Index Terms—Massive multiple-input multiple-output (MIMO), multiple signals classification (MUSIC), multiuser MIMO (MU-MIMO), pilot contamination, time-division duplexing (TDD).

1. Introduction

Recently, there has been a lot of interest in multiuser multiple-input multiple-output (MU-MIMO) wireless communications systems. It has also been verified that by exchanging channel state information (CSI) across the base station (BS) in each cell, the coordinated downlink beamforming technique can be applied to effectively increase the capacity and reliability of a cellular system. This is the concept of network multiple-input multiple-output (MIMO)[1]. Though network MIMO provides the best performance, nevertheless, the information exchange overhead limits its applicability. Fortunately, the complexity of exchanging CSI can be relaxed by simply increasing the number of antennas at each BS transmitter, which is referred to as massive MIMO. This result is rooted in the asymptotics of “random matrix theory”[2]. The channel response vectors associated with different radio terminals (RTs) tend to be nearly orthogonal when the number of BS antennas is large. This makes it possible to reject interference at the BS side by simple linear signal processing approaches, such as matched filter (MF) precoding and maximum ratio combining (MRC). Thereby, the per-cell beamforming scheme can be
used in massive MIMO systems to achieve optimum performance. Moreover, massive MIMO is apt to be included in 5G networks due to the following benefits.

1) Energy efficiency: More antennas in massive MIMO improve the capability to focus the field strength to a specific geographical point. Each single-antenna RT in a massive MIMO system can scale down its transmit power proportionally to the number of antennas at BS.

2) Reliability and robustness: Failure of one or a few of the antennas units would not appreciably affect the system.

3) Reduction of latency: Massive MIMO relies on “the law of large numbers” to make sure that small scale fading averages out when signals from a large number of antennas are combined in the air, so fading no longer limits latency.

Apart from the above benefits, nevertheless, abundant CSI is needed to be estimated in the massive MIMO system. Most past studies have assumed the time-division duplexing (TDD) mode, since it is much more preferred for its channel reciprocity between the uplink and downlink. In TDD, only CSI for the uplink needs to be estimated and can be exploited for downlink beamforming. Ideally, every RT in a massive MIMO system is assigned an orthogonal uplink pilot sequence for channel estimation. However, the effect of reusing pilot sequences leads to negative consequences, which is referred to as pilot contamination. It is well known that channel estimation and cell throughput are plagued by the pilot contamination effects. In the studies of [9] and [10], an eigenvalue decomposition (EVD) based subspace method was proposed, however, to resolve the scalar multiplicative ambiguity between the estimated and actual channel vectors, a short training sequence is still required. A time-shifted pilot sequence method was proposed in [6] in order to avoid sending correlated (non-orthogonal) pilot sequences simultaneously. However, stringent timing control is required and it becomes impractical in a dense cellular system. Similar to the concept of [6], asynchronized pilot transmission was proposed in [11] and [12] to eliminate the correlation of the channel estimation error. To reduce the pilot contamination effect, a pilot reuse scheme was considered. Using the Bayesian channel estimation, the work of [14] found that pilot contamination can disappear under the condition that the channel angular spreads of active users do not overlap. It was shown in [15] and [16] that pilot contamination does not occur with power control and power-controlled handoff when the proposed nonlinear channel estimator is used. In the work of [17], a pilot decontamination method was proposed in a massive MIMO system by exploiting channel sparsity.

This paper considers a multiuser multi-cell massive MIMO system, in which a TDD mode is assumed and BS estimates each RT’s instantaneous CSI during the uplink transmission. Due to the reuse of pilot sequences in each cell, we need to perform channel estimation in the presence of pilot contamination. We propose a multiple signals classification (MUSIC)-based channel estimation method in which the pilot contamination effect can be completely removed as the array size of BS grows large. The rationale is based on the theory proposed in [14] that multipath components of interfering signals with angle-of-arrivals (AOAs) outside the AOA region for the desired user will tend to fall in the null space of its covariance matrix as the array size of BS is large. In what follows, in the uplink pilot transmission phase, we first propose a pilot sequence reuse pattern by carefully assigning an identical pilot sequence to the user in each cell with non-overlapping AOA. There then, we exploit the fact that the received signal strength at the intended cell is larger than the strength of the interfering signal coming from the adjacent cell. Therefore, we are able to distinguish the signal subspace from the interference subspace at each BS. And the signal subspace becomes orthogonal to the interference subspace. Then, a projection of the received signal onto the signal subspace would remove interference from adjacent cells and complete removal of pilot contamination can be attained. An extensive analysis on the impact of the limited array size as well as inevitable estimation error of the correlation matrix on system performance is also provided in this paper.
The rest of this paper is organized as follows. In Section 2, we describe the system model and formulate the problem. Section 3 depicts the pilot contamination problem and the rationale of the MUSIC-based channel estimation algorithm. In Section 4, we demonstrate the system performance and discuss the numerical results. Concluding remarks are finally made in Section 5.

2. System and Channel Model

In this paper, we consider a cellular network of $L$ cells, with full spectrum reuse. Each cell has $K$ served single-antenna RTs and one BS with $M$ antennas. Denote the channel coefficient from the $k$th user in the $l$th cell to the $m$th antenna of the intended BS as $g_{k,l,m}$, which is equal to a complex small-scale fading factor times an amplitude factor that accounts for geometric attenuation and large-scale fading:

$$ g_{k,l,m} = \sqrt{\beta_{k,l}} h_{k,l,m} $$  \hspace{1cm} (1)

where $h_{k,l,m}$ is the fast (small-scale) fading coefficient; $\sqrt{\beta_{k,l}}$ represents the geometric attenuation and shadow (large-scale) fading, which are assumed to be well known and independent of the antenna index $m$. Therefore, the considered MIMO system is consisted of multiple multiple-input single-output (MISO) systems with an $M$-by-$1$ channel vector $\{g_k\}_{k=1,2,\ldots,K}$, where $g_k \triangleq [g_{k,1,1}, g_{k,1,2}, \ldots, g_{k,1,M}]^T$ denotes the channel vector between BS of the intended cell and the $k$th user on the $l$th cell. Here $\triangleq$ is used for “is defined as” and $[\cdot]^T$ stands for the transpose of a matrix or vector. We may also regard it as the spatial signature for specific $(k, l)$.

It should be noted that the spatial signature is unique for each user as long as their physical locations are different. Based on (1) and $h_k \triangleq [h_{k,1,1}, h_{k,1,2}, \ldots, h_{k,1,M}]^T$, we have

$$ g_k = \sqrt{\beta_k} h_k. $$  \hspace{1cm} (2)

Furthermore, we assume that BS in each cell is equipped with a uniform linear array (ULA) with antenna spacing less than or equal to a half wavelength. Hence we have the following multipath model:\[267]\[268\]:

$$ h_{k,l} = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} a_{k,l,p} a_{k,l,p} $$  \hspace{1cm} (3)

where $P$ is the arbitrary number of the independent and identically distributed (i.i.d.) paths; $a_{k,l,p} \sim \mathcal{CN}\{0, \sigma^2_{a_{k,l}}\}$ is independent over the path index $p$, where $\sigma^2_{a_{k,l}}$ is the $(k, l)$th channel’s average attenuation; $a_{k,l,p}$ is the steering (response) vector of ULA at the intended BS with respect to the signal coming from the $p$th path. It can be represented by the Vandermonde form:

$$ a_{k,l,p} \triangleq \frac{1}{\sqrt{M}} \begin{bmatrix} 1 \\ e^{j \frac{2\pi d}{\lambda} \cos(\theta_{k,l,p})} \\ \vdots \\ e^{j \frac{2\pi (M-1)d}{\lambda} \cos(\theta_{k,l,p})} \end{bmatrix} $$  \hspace{1cm} (4)

where $d$ is the antenna spacing at BS and $\lambda$ is the signal wavelength. Without loss of generality, we set $d=\lambda/2$ throughout this paper. $\theta_{k,l,p} \in [0, \pi]$ is AOA emitted from the $k$th user’s $p$th path in the $l$th cell. Upon defining

$$ \{A_k\} \triangleq [a_{k,1,1}, a_{k,1,2}, \ldots, a_{k,1,M}] $$

$$ \{a_k\} \triangleq [a_{k,1,1}, a_{k,1,2}, \ldots, a_{k,1,M}]^T $$

$$ \{\alpha_k\} \triangleq [\alpha_{k,1,1}, \alpha_{k,1,2}, \ldots, \alpha_{k,1,M}]^T $$

$$ \{\beta_k\} \triangleq [\beta_{k,1,1}, \beta_{k,1,2}, \ldots, \beta_{k,1,M}]^T $$

$$ \{\sigma^2_{a_{k,l}}\} \triangleq [\sigma^2_{a_{k,1,1}}, \sigma^2_{a_{k,1,2}}, \ldots, \sigma^2_{a_{k,1,M}}] $$

$$ \{\theta_{k,l,p}\} \triangleq [\theta_{k,1,1}, \theta_{k,1,2}, \ldots, \theta_{k,1,M}] $$

$$ \{\theta_{k,l,p}\} \triangleq [\theta_{k,1,1}, \theta_{k,1,2}, \ldots, \theta_{k,1,M}]^T $$

$$ \{\gamma_k\} \triangleq [\gamma_{k,1,1}, \gamma_{k,1,2}, \ldots, \gamma_{k,1,M}]^T $$
can be written in the compact form:

$$h_{k,i} = \frac{1}{\sqrt{P}} A_{k,i} \alpha_{k,i}$$  \hspace{1cm} (5)$$

where $E\{a_j a_j^H\} = \sigma^2 n_j I_p$, with $E\{\cdot\}$ used for the expectation, $\cdot^H$ for the complex transpose of a matrix or vector, and $I_p$ being an identity matrix with the size of $p$.

3. Channel Estimation

3.1. Pilot Contamination

We consider here a TDD scheme, in which reciprocity between the uplink and downlink channels can be assumed. For TDD-based massive MIMO transmission systems, pilot sequences are transmitted from RTs in the uplink phase to estimate channels. There are $L$ time-synchronized cells in the considered model. Ideally, the pilot sequences employed by all RTs should be orthogonal. However, the number of orthogonal pilot sequences with a given period and bandwidth is limited, which in turn leads to pilot contamination. We denote the pilot sequences, with the length of $\tau$, used by single-antenna RTs within the same cell by

$$s_k \doteq \begin{bmatrix} s_{k,1}, s_{k,2}, \ldots, s_{k,\tau} \end{bmatrix}^T, \quad k = 1, 2, \ldots, K.$$  \hspace{1cm} (6)

These pilot signals transmitted by each RT within a cell are mutually orthogonal such that

$$s_k^T s_j = \tau \delta(k-j) = \begin{cases} \tau, & k = j \\ 0, & k \neq j. \end{cases}$$  \hspace{1cm} (7)

The same set of orthogonal pilot sequences, $\{s_k\}_{k=1,2,\ldots,K}$, is reused in all the remaining cells. As a result, intra-cell interference is negligible in the channel estimation phase. However, identical pilot sequences assigned to users in other cells result in inter-cell interference. During the uplink pilot phase, all users simultaneously transmit pilots to their BSs. Hence, the received signal matrix at the target BS antenna array can be expressed as

$$Y = \sqrt{p'} \sum_{k=1}^{K} \sqrt{\beta_k} h_{k,1} s_k^T + N$$  \hspace{1cm} (8)

where $Y, N \in \mathbb{C}^{M \times \tau}$, $N$ is the additive white gaussian noise (AWGN) vector at BS, whose entries are i.i.d. circular complex Gaussian random variables with zero mean and variance $\sigma^2$. $p'$ is the uplink transmit power. Without loss of generality, we let Cell 1’s first RT be the target RT. To estimate the channel vector, $h_{1,1}$, BS of Cell 1 correlates its received signal with $s_1$, which yields

$$y_1 = \frac{Y s_1}{\tau} = \frac{1}{\tau} \left( \sqrt{p'} \sum_{k=1}^{L} \sqrt{\beta_k} h_{k,1} s_k^T + N \right) s_1 = \sqrt{p'} \sum_{k=1}^{L} \sqrt{\beta_{1,k}} h_{1,k} + n_1 = \sqrt{p'} \sqrt{\beta_{1,1}} h_{1,1} + \sqrt{p'} \sum_{k=2}^{L} \sqrt{\beta_{1,k}} h_{1,k} + n_1$$  \hspace{1cm} (9)

where $\{h_{1,k}\}_{k=2,3,\ldots,L}$ denotes the channel vector between BS in Cell 1 and RT in the $k$th cell that transmits the pilot sequence of $s_k$. In writing (9), we have used the pilot sequences’ orthogonal property of (7). The noise vector $n_1 = N s_1/\tau$ is still Gaussian with zero mean and covariance matrix $(\sigma^2/\tau)I_M$. In what follows, the estimate of $h_{1,1}$ can be simply obtained as

$$h_{1,1} = \frac{Y s_1}{\tau \sqrt{\beta_{1,1}}} = \sqrt{p'} h_{1,1} + \sqrt{p'} \sum_{k=2}^{L} \sqrt{\beta_{1,k}} h_{1,k} + n_1 \sqrt{\beta_{1,1}}.$$  \hspace{1cm} (10)
As revealed in (10), $\mathbf{h}_{1,1}$ is a linear combination of the channel vectors of the users in the remaining $L-1$ cells using the same pilot sequence $\mathbf{s}_i$. The estimation error arisen from the second term is the consequence of pilot contamination. This interference, unlike intra-cell interference, will not disappear as the number of BS antennas increases. We illustrate the pilot contamination concept in Fig. 1, where 3 cells are considered and only RTs with the same pilot sequence $\mathbf{s}_i$ are shown. Assume RT in Cell 1 is intended, we use solid and dash lines to represent the desired and interfering signals, respectively. As we can observe from Fig. 1, RTs in Cell 2 and Cell 3 interfere with BS in Cell 1 during uplink transmission. Consequently, BS in Cell 1 not only sends the beamforming signals to its own RT, but also to RTs in Cell 2 and Cell 3 during the downlink transmission, and therefore creates a strong source of directional interference.

In the downlink transmission phase, if we employ the channel estimate, $\mathbf{\hat{h}}_{1,1}$, as the downlink beamforming vector for the first user in Cell 1, then

$$\mathbf{w}_{1,1} = \alpha_{1,1} \mathbf{\hat{h}}_{1,1}$$

where $\alpha_{1,1}$ is the power normalization factor. Then BS transmits an $M$-dimensional downlink beamforming vector to $K$ users, which yields

$$\mathbf{x}_1 = \sqrt{p_d} \sum_{k=1}^{K} \mathbf{w}_{k,1} s_{k,1}'$$

where $p_d$ is the downlink transmit power; $\mathbf{w}_{k,1}$ is the $M$-by-$1$ downlink beamforming vector that carries the $k$th RT’s downlink data bit $s_{k,1}'$. It is evident that channel estimation caused by pilot contamination severely degrades the downlink transmission performance in two aspects:

1) The received signal at the desired RT contains interference terms

$$\sqrt{\beta_{1,1}} \sum_{l=2}^{L} \sqrt{\beta_{1,l}} \mathbf{h}_{1,l}^H \mathbf{h}_{1,1}.$$ 

2) If we use (10) as the downlink beamforming vector, the second term, $\sqrt{\beta_{1,1}} \sum_{l=2}^{L} \sqrt{\beta_{1,l}} \mathbf{h}_{1,l}^H \mathbf{h}_{1,1}$, will definitely interfere the reception of RTs in the remaining $L-1$ cells using the same pilot sequence. Similarly, the desired RT should be interfered by the downlink transmission of the remaining $L-1$ cells.

From the law of large numbers, it follows that the channel vectors between the users and BS become pairwisely orthogonal\(^\mathrm{i}\), i.e.,

$$\frac{1}{M} \mathbf{h}_{k,l}^H \mathbf{h}_{k,l'} = \delta(k-k', l-l').$$

The first problem may be neglected according to (13) as $M$ grows large, which is usually the case in massive MIMO. However, the second problem is inevitable and will severely degrade the performance as $L$ is large.
3.2. MUSIC-Based Pilot Decontamination and Channel Estimation

In this section, we aim at proposing a pilot decontamination method by taking advantage of the multiple antenna dimensions. Exploiting (5), we first rewrite (9) as

$$y_l = \frac{\sqrt{P}}{T} \sum_{i=1}^{L} \sqrt{\beta_i} h_{1,i} + n_l = \frac{\sqrt{P}}{T} \sum_{i=1}^{L} \sqrt{\beta_i} A_{1,i} \alpha_{1,i} + n_1. \quad (14)$$

The covariance (correlation) matrix of the observed data vector can be obtained as

$$R_{y_l} \triangleq E[y_l y_l^H] = \frac{P}{T} \left( \beta_{1,l} R_{1,l} + \sum_{i=2}^{L} \beta_i R_{i,l} \right) + \frac{\sigma^2}{T} I_M$$

where

$$R_{w_l} \triangleq E[h_{w_l} h_{w_l}^H] = E[A_{w_l} A_{w_l}^H] A_{w_l}^H]. \quad (16)$$

In massive MIMO, the number of BS antennas is on the order of or more than 100. In the scenario of a large-number-of-antennas case, it has been proven in [14] that multipath components with AOAs outside the AOAs region for a given user will tend to fall in the null space of its covariance matrix. Denote $\theta^{\min}_{l,i}$ and $\theta^{\max}_{l,i}$ be the minimum and maximum AOAs of the $k$th RT in the $l$th cell, then based on the result in [14], we have that if the $L-1$ intervals, $[\theta^{\min}_{i,l}, \theta^{\max}_{i,l}]$ with $l = 2, 3, \ldots, L$, are strictly non-overlapping with the desired channel’s AOA interval, $[\theta_{\min}^{\min}, \theta_{\max}^{\max}]$, satisfies

$$\text{null}(A_{1,1}) \supset \text{span}\{A_{1,2}, A_{1,3}, \ldots, A_{1,l} \}. \quad (17)$$

In this paper, we assume that the base station controller (BSC) or mobile switching center (MSC) knows the positions of all RTs in each cell and the AOA spread of multipath components is confined to a region of space. Thereby, it is up to our disposal that BSC assigns the same pilot sequence, e.g., $s_i$, to the specific RT in each cell such that the selected RTs exhibit multipath AOAs that do not overlap with AOAs for the desired user.

Performing EVD on $R_{y_l}$, we have

$$R_{y_l} = U \Sigma U^H \quad (18)$$

where $\Sigma = \text{diag} \{ \lambda_1, \lambda_2, \ldots, \lambda_M \}$ is a diagonal matrix with eigenvalues arranged in a descending order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M$. $U = [u_1, u_2, \ldots, u_M]$ is the eigenvector matrix in which $u_i$ corresponds to $\lambda_i$. Due to the pass loss effect, we have that $\beta_{1,1} > \beta_{1,i}$ with $l = 2, 3, \ldots, L$, then we may separate $U$ into signal subspace, span $\{u_1, u_2, \ldots, u_P\}$, and noise subspace, span $\{u_{P+1}, u_{P+2}, \ldots, u_M\}$. From the matrix theory, we have that the column space of $A_{1,1}$ spans the signal subspace.

$$\text{CSP}\{A_{1,1}\} = \text{span}\{a_{1,1,1}, a_{1,1,2}, \ldots, a_{1,1,p}\} = \text{span}\{u_1, u_2, \ldots, u_P\}. \quad (19)$$

Since the signal subspace and noise subspace are orthogonal complements, upon defining the noise projection matrix:

$$P_{H} \triangleq \sum_{i=1}^{M} u_i u_i^H \quad (20)$$

it is evident that the projection of each signal vector on $P_{H}$ must be zero.

$$P_{H} a_{1,1,p} = 0, \quad \forall \quad p = 1, 2, \ldots, P. \quad (21)$$

For $d=\lambda/2$, to estimate $\{a_{1,1,p}\}_{p=1, 2, \ldots, P}$, we first define

$$a(\theta) \triangleq [1, e^{-j\pi\cos\theta}, \ldots, e^{-j[p(M-1)\cos\theta]}]^T. \quad (22)$$
The rationale of the MUSIC\textsuperscript{[9]} algorithm is to exploit the fact as depicted in (21). AOAs of the desired RT can be found by searching over the $\theta$ domain for $P$ peaks (ideally extremely high) of the MUSIC (spatial) spectrum.

$$S_{\text{MUSIC}}(\theta) = \frac{1}{a^H(\theta)P_*a(\theta)}. \quad (23)$$

A distinct feature of ULA is that the steering vector has a Vandermonde structure, therefore we may first substitute $z=e^{j\theta \text{cos} \phi}$ in (22) to obtain $a(z) = \left[1, z, \cdots, z^{M-1}\right]^T$, and then the spatial spectrum (denominator of (23)) can be converted into a polynomial with the degree of $2M-1$:

$$S(z) = a^H(z)P_*a(z) = a^T(z^{-1})P_*a(z). \quad (24)$$

In what follows, the spatial spectrum search problem is equivalent to the polynomial rooting problem. This is referred to as the Root-MUSIC algorithm\textsuperscript{[99]}. By finding $P$ roots of $S(z)$ on the unit circle of the $z$-plane, \{\(z_1, z_2, \cdots, z_P\)\}, AOAs of the desired RT can be calculated by

$$\hat{\theta}_{1,i} = \cos^{-1}\left(\frac{-1}{|z_i|}\right), \quad i = 1, 2, \cdots, P. \quad (25)$$

Substituting (25) into (4), the steering vectors, \(\{\hat{a}_{1,1,p}\}_{p=1,2,\cdots,P}\), can be obtained.

Note that in a practical situation, $R_y$ is unknown and needs to be estimated. Under the ergodic assumption, we collect a sequence of the measurements, \(\{y_i^{(n)}\}_{n=1,2,\cdots,N}\), and perform time average instead of ensemble average. Hence, the estimate of $R_y$ is given by

$$\hat{R}_y = \frac{1}{N} \sum_{n=1}^{N} y_i^{(n)} y_i^{(n)H}. \quad (26)$$

It should be noted that the correlation matrix of the observed data vector is estimated by sample average with the size of $N$. The estimation error, which is due to the use of a finite window size, will decrease as the number of samples $N$ increases (this requires that the statistics is invariant during the observation interval).

The algorithm of the proposed MUSIC based channel estimation algorithm is proceeded in the following.

Step 1: BSC allocates each pilot sequence, e.g., $s_i$, to the specific RT (one in each cell) such that the selected RTs exhibit multipath AOAs that do not overlap with AOAs for the desired RT.

Step 2: The intended BS correlates its received signal on $s_i$ to remove intra-cell interference as well as most of the inter-cell interference (except the users in other cells using the same pilot sequence).

Step 3: The intended BS collects $N$ data vectors, \(\{y_i^{(n)}\}_{n=1,2,\cdots,N}\), and uses (26) to compute $\hat{R}_y$.

Step 4: Perform EVD on $\hat{R}_y$, we obtain eigenvalues (in a descending order) $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$, and the corresponding eigenvectors \(\{u_1, u_2, \cdots, u_M\}\).

Step 5: Construct the noise projection matrix by (20),

$$\hat{P}_N = \sum_{j=P+1}^{M} u_j u_j^H.$$  

Step 6: Compute the MUSIC spectrum by (23),

$$\hat{S}_{\text{MUSIC}}(\theta) = \frac{1}{a^H(\theta)\hat{P}_N a(\theta)}.$$  

Step 7: Find $P$ peaks from $\hat{S}_{\text{MUSIC}}(\theta)$ to get the AOA estimate, \(\hat{\theta}_{1,1,p}\)\(p=1,2,\cdots,P\). And the steering vectors, \(\{\hat{a}_{1,1,p}(\hat{\theta}_{1,1,p})\}_{p=1,2,\cdots,P}\), can be obtained according to (4).
As soon as \( \{ \hat{\mathbf{a}}_{1,1,p}(\hat{\theta}_{1,1,p}) \}_{p=1,2,\ldots,P} \) has been obtained, the pilot transmission as well as channel estimation phase is completed. In what follows, the channel estimate of \( \mathbf{h}_{1,1} \) can be obtained as an equal gain combination of each path’s steering vector.

\[
\mathbf{h}_{1,1} = \sum_{p=1}^{P} \hat{\mathbf{a}}_{1,1,p}(\hat{\theta}_{1,1,p}). 
\]  

(27)

As we compare (27) with (10), it is evident that pilot contamination has been mitigated. The downlink beamforming vector can be constructed as

\[
\mathbf{w}_{1,1} = \frac{1}{\sqrt{\beta_{1,1}}} \mathbf{h}_{1,1} = \frac{1}{\sqrt{\beta_{1,1}}} \sum_{p=1}^{P} \hat{\mathbf{a}}_{1,1,p}(\hat{\theta}_{1,1,p}). 
\]  

(28)

4. Performance Evaluation

In this section, we aim at evaluating the system performance through computer simulations. Unless otherwise mentioned, we set the total number of cells to be \( L=3 \). The number of paths for each RT is \( P=5 \), the path gain is generated by \( a_{i,p} \sim \mathcal{CN}(0,1) \) and is independent over the path index \( p \). AOA of each path emitted from the desired user is within \( (0.21\pi, 0.25\pi) \) and AOAs of interfering users are non-overlapping with AOA of the desired RT. The large-scale fading coefficients for the intended cell and interfering cells are set to be \( \beta_{1,1} = 0.9 \), \( \beta_{1,2} = 0.5 \), and \( \beta_{1,3} = 0.4 \), respectively. The uplink transmit power is set to be \( P_u/N_0 = 15 \) dB, where \( N_0 \) represents the one-sided power spectral density of the background noise. The result in each simulation example is obtained from the average of 500 independent trials.

In the first simulation example, we provide three different sample sizes, \( N=100, 300, \) and \( 500 \), to estimate the correlation matrix, \( \hat{\mathbf{R}}_y \), and the resulted MUSIC spectra are presented in Fig. 2. As depicted in Fig. 2, we scan \( S_{MUSIC}(\theta) \) over 1000 points from \( \theta=0 \) to \( \theta=\pi \), and there are exactly 5 peaks. The 5 peaks in \( S_{MUSIC}(\theta) \) reveal AOAs \( \{\hat{\theta}_{1,1,p}\}_{p=1,2,\ldots,P} \) of the desired RT. As expected, higher peaks can be obtained as we increase the sample size. It is also verified that \( N=100 \) is enough to accurately estimate AOAs. Please note that ideally, we search for “infinitely” high spectral peaks. However, in practice, due to the estimation error arisen from a limited sample size, the MUSIC spectrum does not exhibit infinitely high peaks.

Fig. 2. MUSIC spectra in terms of different sample sizes.
As described in Section 3, the proposed algorithm relies on the criterion that multipath components with AOAs outside the AOAs region for a given RT will tend to fall in the null space of its covariance matrix in the scenario of the large-number-of-antennas case. The main focus of the second simulation example is to examine the array size required to meet this characteristic. In Fig. 3, we examine $\hat{S}_{\text{MUSIC}}(\theta)$ for $M=400, 600$, and $800$, respectively. The sample size for the estimation of $R_y$ is set to be $N=500$. As we can observe from Fig. 3, only when $M=400$, there is one false peak near $\theta=0$. As we increase $M$, the 5 AOA peaks are more obvious. One may argue that is $M \geq 400$ a practical choice? It is well known that the millimeter wave (mm wave) is expected to be adopted in 5G due to the fact that the spectrum is less crowded and available bandwidths are broader. Thereby, shorter wavelengths associated with higher frequencies tremendously reduce the size of the antenna array, in which massive MIMO is no more impractical.

![MUSIC spectra in terms of different array sizes.](image)

**Fig. 3.** MUSIC spectra in terms of different array sizes.

5. Conclusions

This paper proposes an MUSIC-based channel estimation method in the context of multiuser multi-cell massive MIMO systems. In the uplink phase, we exploit the appropriate pilot sequences assignment strategy by designating the same pilot sequence to a selected RT in each cell with non-overlapping AOA. There then, taking advantage of the characteristics that the signal subspace is orthogonal to the interference subspaces for a large-scale antenna system, we have proposed an MUSIC-based channel estimator. We have demonstrated analytically that such an approach leads to complete removal of the pilot contamination effect. Computer simulations verified the validity of the proposed channel estimator under an appropriate sample size as well as the array size.

References


WU: MUSIC-Based Pilot Decontamination and Channel Estimation in Multiuser Massive MIMO System


Wei-Chiang Wu was born in Miaoli in 1964. He received the B.S. degree in electrical engineering from Chung Cheng Institute of Technology, Keelung in 1986, and the M.S. and Ph.D. degrees both in electrical engineering from the National Tsing Hua University, Hsinchu in 1992 and 1998, respectively. From 1992 to 1994, he was an assistant researcher with the Communication Department, Chung Shan Institute of Science and Technology, Taoyuan. Since 2011, he has been a professor with the Department of Electrical Engineering, Da-Yeh University, Changhua. His current research interests include multiuser detection, smart antenna technology, cognitive radio and ultra-wideband (UWB) impulse radio (IR) cognitive radio, and massive MIMO technology.