Towards the Energy Efficiency of Resource Allocation Algorithms for OFDMA Downlink MIMO Systems

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Abstract—The problem of the simultaneous multi-user resource allocation algorithm in orthogonal frequency division multiple access (OFDMA) based systems has recently attracted significant interest. However, most studies focus on maximizing the system throughput and spectral efficiency. As the green radio is essential in 5G and future networks, the energy efficiency becomes the major concern. In this paper, we develop four resource allocation schemes in the downlink OFDMA network and the main focus is on analyzing the energy efficiency of these schemes. Specifically, we employ the advanced multi-antenna technology in a multiple input-multiple output (MIMO) system. The first scheme is based on transmit spatial diversity (TSD), in which the vector channel with the highest gain between the base station (BTS) and specific antenna at the remote terminal (RT) is chosen for transmission. The second scheme further employs spatial multiplexing on the MIMO system to enhance the throughput. The space-division multiple-access (SDMA) scheme assigns single subcarrier simultaneously to RTs with pairwise “nearly orthogonal” spatial signatures. In the fourth scheme, we propose to design the transmit beamformers based on the zero-forcing (ZF) criterion such that the multi-user interference (MUI) is completely removed. We analyze the tradeoff between the throughput and power consumption and compare the performance of these schemes in terms of the energy efficiency.

Index Terms—Energy efficiency, multiple input-multiple output (MIMO), multi-user resource allocation, orthogonal frequency division multiple access (OFDMA), space-division multiple-access (SDMA), spatial multiplexing.

1. Introduction

In 4G and future cellular networks, mushrooming users need to share the spectrum to achieve high-rate multimedia communications. However, with the explosive growth of high-rate applications, more and more energy is consumed in wireless networks to ensure the fulfillment of quality-of-service (QoS) requirements. Recently, increasing attention has been paid to energy-efficient communications under the background of limited energy resources and environmental burdens. Therefore, green radio (GR)\(^1\), which emphasizes on the energy efficiency, has been proposed as an effective solution and is becoming the mainstream for the future wireless network design. Orthogonal frequency division multiple access (OFDMA) is employed in 4G long term evolution (LTE)\(^2\) and is expected to be applied in future cellular networks. The rationale for OFDMA is dynamically allocating subcarriers to

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the user with the best channel state. Its benefits include the robust multipath suppression, ability to combat intersymbol interference (ISI), flexibility in accommodating many users with widely varying data rates, and high throughput. Recently, quite a few scheduling and resource (subcarriers) allocation algorithms have been proposed for OFDMA cellular systems\cite{3-7}. Resource allocation is essentially a constrained-optimization problem\cite{8} that either maximizes the overall data rate or minimizes the total transmit power subject to specific constraints. Most past studies emphasize on the ability of providing high throughput as well as spectral efficiency in OFDMA networks, while its energy efficiency, which is measured by the number of bits transmitted per joule of energy consumed, has not drawn much attention. Our focus is mainly on analyzing the energy efficiency of various resource allocation algorithms.

It is well-known that the massive MIMO technique is apt to be included in future 5G networks\cite{9-14}. The key benefits of wireless communications provided by the antenna array include:

1) Spatial diversity. It can effectively combat fading.
2) Interference suppression. The interference can be removed from the desired user as long as the array size exceeds the number of interferers.
3) Spatial multiplexing. It allows different signal sources to be sent simultaneously in the same bandwidth.

All the above techniques can effectively increase the capacity or throughput of a wireless communications system. The aim of this paper is to jointly exploit the three benefits of the multi-antenna technology to develop a simple yet efficient resource allocation algorithm in the downlink of an OFDMA-based cellular system.

This paper considers a multi-user MIMO system, in which each cell consists of an $L$-antennas base station (BTS) and $K$ remote terminals (RTs), each equipped with $Q$ antennas. The downlink OFDMA system with the assumption that each RT estimates and feedbacks the instantaneous channel state information (CSI) to BTS. BTS controls its transmit power for each user based on CSI and the purpose of power control is to transmit enough power so that the system performance can achieve required QoS, in which the bit error rate (BER) is the performance metric. We develop four resource allocation schemes, which allocate subcarriers to each user via various array processing techniques and subject to the constraint that each user has a minimum data rate requirement.

The first scheme is based on transmit spatial diversity (TSD), in which BTS designates the subcarrier to the user with the highest vector channel gain and employs transmit beamforming to further enhance the received signal energy. The second scheme exploits the spatial multiplexing (SM) technique on the MIMO system between BTS and a specific RT. Since multiple substreams can be sent through the MIMO system, the data rate is expected to be increased. Each subcarrier can only be assigned to a single user at a specific time for the above two schemes. While in the third scheme, we designate the subcarrier to the users with “nearly orthogonal” spatial signatures. We refer to it as the space-division multiple-access (SDMA) scheme. Though the throughput of the SDMA scheme is expected to be increased, nevertheless, an obvious flaw of this scheme is that spatial channels are rarely orthogonal in practice. In the fourth scheme, we propose to design prefilters at BTS that meets the zero-forcing (ZF) criterion to remove multi-user interference (MUI). Since MUI is removed and each subcarrier can be assigned simultaneously to all the user terminals, therefore throughput multiplication can be achieved. However, in the case of $L < KQ$, complete MUI suppression is not possible. We propose to select out of spatial signatures that contribute the maximum throughput to the system. We call it as the selected ZF (SZF) scheme. Using the utility function as defined in \cite{15}, which is measured by the ratio of throughput over transmit power, as the performance metric, we are able to compare the performance of these schemes in terms of the energy efficiency. Moreover, we study the tradeoff between the throughput and power consumption.
In the rest of this paper, we use the upper and lower case boldface letters to denote matrices and vectors, respectively. $A^\top$ and $A^\dagger$ stand for the matrix or vector transpose and complex transpose, respectively. We will use $E\{\cdot\}$ for expectation (ensemble average), $|\cdot|$ for vector norm, and $\equiv$ for “is defined as”. $\|A\|_F$ denotes the Frobenius norm of matrix $A$. $I_k$ denotes an identity matrix with a size of $K$. $e_k$ denotes the $k$th column vector of an identity matrix. A complex normal variable with mean $\mu$ and variance $\sigma^2$ reads as $\mathcal{CN}(\mu, \sigma^2)$. $\bar{x}$ denotes the complex conjugate of $x$. $\text{tr}(A)$ denotes the trace (sum of the diagonal element) of the square matrix $A$.

2. System Model

Consider the downlink of an OFDMA system, where in a specific cell, BTS is equipped with $L$ antennas and there are $K$ RTs, each with $Q$ antennas. For each subcarrier, the flat-fading channel model is assumed. Let $\{h_{n,k,l,q}\}_{q=1,2,\ldots,Q}^{l=1,2,\ldots,L}$ be the instantaneous channel gain between the $k$th RT’s $q$th and $l$th antennas of BTS at the $n$th subcarrier, which includes the effects of path loss, shadowing, and fading. For each subcarrier $n$, it can also be regarded as a multiple MIMO system with the channel matrix $\{H_k^n\}_{k=1,2,\ldots,K}$ given by

$$
H_k^n \equiv \begin{bmatrix}
    h_{k,1,1}^n & h_{k,1,2}^n & \cdots & h_{k,1,Q}^n \\
    h_{k,2,1}^n & h_{k,2,2}^n & \cdots & h_{k,2,Q}^n \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{k,L,1}^n & h_{k,L,2}^n & \cdots & h_{k,L,Q}^n
\end{bmatrix}_{L\times Q}
$$

(1)

where $\{h_{k,q}\}_{q=1,2,\ldots,Q}$ denotes the $L$-by-$1$ channel vector seen by the $k$th user’s $q$th antenna.

$$
h_{k,q}^n \equiv \begin{bmatrix}
    h_{k,1,q}^n & h_{k,2,q}^n & \cdots & h_{k,L,q}^n
\end{bmatrix}^\top.
$$

(2)

Let us consider here a time-division duplexing (TDD) scheme, in which reciprocity between the uplink and downlink channels can be assumed. Consequently, according to the method proposed in [12], BTS can estimate CSI from the uplink pilot signals transmitted by each user. A schematic illustration of the system under consideration is depicted in Fig. 1.

Fig. 1. Block diagram of the multi-user MIMO system.
For simplicity, we invoke the following assumptions throughout this paper:
1) BTS has perfect knowledge of CSI, which is periodically reported by each user.
2) CSI stays constant during the resource allocation process.

Since all RTs share the same bandwidth, we attempt to develop an energy-efficient resource allocation algorithm to allocate \( N \) subcarriers to the \( K \) users subject to the pre-determined constraints. A plausible utility function is used to measure the number of reliable bits transmitted per Joule (J) of energy consumed\(^{[15]}\):

\[
U = \frac{T}{P} \text{ (bits/J)}
\]

where the throughput, \( T \) is defined as the net number of information bits that are transmitted without any error per unit time. \( P \) is the transmit power. The design goal is to maximize \( U \) subject to the following constraints.

1) QoS for each RT should meet the system requirements. In this paper, QoS is measured by BER.
2) The information bits allocated for the \( k \)th user per unit time should meet a minimum rate requirement.

Please note that the resource allocation algorithms described in this paper are flexible with every attempt to maximize the utility function, thereby, the subcarriers allocated for each user are distributed rather than bunched.


This paper employs the binary phase shift keying (BPSK) modulation, though the extension to other modulation techniques is straightforward. BER for the BPSK modulation is\(^{[16]}\)

\[
P_{e;BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\]

where \( E_b \) denotes the received bit energy, \( N_0 \) is the one-sided power spectral density of additive white Gaussian noise (AWGN). \( Q(x) \equiv \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt \) is the complementary cumulative distribution function of a standard normal random variable. Let \( R \) be the transmission rate (the transmitted bits per unit time), then the throughput can be obtained as

\[
T = R \left(1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right).
\]

To maintain QoS, it is usually required that BER at each RT must be lower than a pre-determined threshold, \( P_{e;BPSK} \leq \lambda \). Using (4) and after some manipulations, we have

\[
E_b \geq \frac{N_0}{2} \left(Q^{-1}(\lambda)\right)^2.
\]

The algorithms proposed in this paper are to transmit power as low as possible for achieving required QoS, while maximize the overall throughput. The optimum solution requires an exhaustive search. The number of iterations needed in a single antenna scenario (\( M=Q=1 \)) is about \( O\left(K^N\right) \), which is infeasible even for moderate \( N \) and \( K \). Therefore, the suboptimum solutions are attempted to develop.

3.1. TSD Scheme

In the TSD scheme, each subcarrier can only be assigned to a single user. The transmit power to maintain target QoS is low as long as the channel is good. On the other hand, high power is required when and where the channel is in poor condition. Regard \( \{\mathbf{h}_{k,q}\}_{q=1,2,…,Q} \) as \( K \times Q \) separate channels, and the \( r \)th subcarrier is assigned to the channel with the best gain.
\[(k^*, q^*) = \arg \max_{(k, q) \in \{1, 2, \ldots, K, 1, 2, \ldots, Q\}} \| \mathbf{h}_{k,q}^n \|. \]  

(7)

As revealed by (7), the data that are modulated onto the nth subcarrier are sent from BTS to the qth antenna of the kth RT. Let \( \{ \rho_{k,q}^n \} \) be the subcarrier allocation index, that is, \( \rho_{k,q}^n = 1 \) when the nth subcarrier is assigned to the kth RT’s qth antenna, \( \rho_{k,q}^n = 0 \), otherwise. Thereby, in the TSD scheme, if \( \rho_{k,q}^n = 1 \), then \( \rho_{k,q}^n = 0 \) for all \((k', q') + (k, q)\).

(8)

In the TSD scheme, BTS performs transmit beamforming by matched-filtering with the channel vector. Hence, the weight vector is designed as the complex conjugate of \( \mathbf{h}_{k,q}^n \):

\[ \mathbf{w}_{k,q}^n = \eta_{k,q}^n \mathbf{h}_{k,q}^n \]  

(9)

where \( \eta_{k,q}^n \) denotes the power normalization factor in order to meet QoS at the receiver end, \( \mathbf{w}_{k,q}^n \) is the L-by-1 transmit weight vector that carries the bit \( s_{k,q}^n \). Let the bit energy be normalized to be 1, i.e., \( E \left[ |s_{k,q}^n|^2 \right] = 1 \), therefore, the total transmit power of the TSD scheme can be calculated as

\[ P_{\text{TSD}} = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n E \left[ |s_{k,q}^n|^2 \right] \| \mathbf{w}_{k,q}^n \|^2 = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n (\eta_{k,q}^n)^2 \| \mathbf{h}_{k,q}^n \|^2. \]  

(10)

The received signal at the kth RT’s qth antenna yields

\[ y_{k,q}^n = s_{k,q}^n \mathbf{h}_{k,q}^n \mathbf{w}_{k,q}^n + n_{k,q}^n = \eta_{k,q}^n s_{k,q}^n \| \mathbf{h}_{k,q}^n \|^2 + n_{k,q}^n \]  

(11)

where \( n_{k,q}^n \) denotes AWGN with one-sided power spectral density \( N_0 \). Then exploit (11) to calculate BER with \( E_b \) given by

\[ E_b = E \left[ (\eta_{k,q}^n s_{k,q}^n \| \mathbf{h}_{k,q}^n \|^2)^2 \right] = (\eta_{k,q}^n)^2 \| \mathbf{h}_{k,q}^n \|^4. \]  

(12)

Substituting (12) into (6), the power normalization factor of the TSD scheme can be obtained as

\[ \eta_{k,q}^n = \frac{Q^{-1}(\lambda)}{\| \mathbf{h}_{k,q}^n \|} \sqrt{\frac{N_0}{2}}. \]  

(13)

Consequently, we obtain the minimum power required to maintain QoS

\[ P_{\text{TSD}} \geq \frac{N_0}{2} \left( Q^{-1}(\lambda) \right)^2 \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n \| \mathbf{h}_{k,q}^n \|^2. \]  

(14)

The throughput for the kth user can be written as

\[ T_k = (1 - \lambda) \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n; \quad k = 1, 2, \ldots, K. \]  

(15)

And the overall throughput is \( T = \sum_{k=1}^{K} T_k \). From (14) and (15), we can write the utility function as

\[ U_{\text{TSD}} = \frac{T}{P_{\text{TSD}}} \leq \frac{(1 - \lambda) \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n}{\frac{N_0}{2} \left( Q^{-1}(\lambda) \right)^2 \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n \| \mathbf{h}_{k,q}^n \|^2}. \]  

(16)
The power-efficient subcarriers allocation for the TSD scheme is equivalent to finding \( \{\rho_{k,q}^{\circ}\}_{k=1,2,\ldots,K} \) to satisfy the following constrained-optimization problem \( \arg \max_{\rho_{k,q}^{\circ}} U_{\text{TSD}} \).

Subject to:

1) \[
R_k = \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^{\circ} \geq R_{k,\text{min}}
\]
where \( k = 1, 2, \ldots, K \)
2) If \( \rho_{k,q}^{\circ} = 1 \), then \( \rho_{k',q'}^{\circ} = 0 \) for all \( (k', q') \neq (k, q) \).
Hence, it can be obtained
\[
\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^{\circ} = N.
\]

Note that (17) ensures the transmission rate for each user is larger than its minimum rate constraint. And (18) ensures that each subcarrier can only be assigned to a single user.

The algorithm of the proposed TSD scheme is proceeded in the following steps.
1) Starting from the first subcarrier, assign the subcarrier to the vector channel with the highest gain in that subchannel. Let \( W \) be the set, \( W = \{(k, q)\}_{k=1,2,\ldots,K} \), then \( (k^*, q^*) = \arg \max_{(k,q)\in W} ||h_{k,q}^{\circ}|| \).
As soon as subcarrier \( n \) has been assigned to user \( k^* \), set \( \rho_{k^*,q^*}^{\circ} = 1 \) and \( \rho_{k,q}^{\circ} = 0 \) for all \( (k^*, q^*) \neq (k^*, q^*) \).
2) Calculate the cumulative transmission rate by
\[
R_{k^*} = \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k^*,q^*}^{\circ}.
\]
3) Check whether \( R_{k^*} \) meets the minimum bit rate constraint. If \( R_{k^*} < R_{k^*,\text{min}} \), go back to step 1) and check for the next subcarrier. On the other hand, if \( R_{k^*} \geq R_{k^*,\text{min}} \), temporarily exclude user \( k^* \) from the set of \( W \)
\[
W \setminus \{(k^*, q)\}_{k^*,q^*} = \{(k, q)\}_{k=1,2,\ldots,K, q=1,2,\ldots,Q, k \neq k^*}
\]
and go back to step 1) to assign the subchannels to those users that have not achieved the minimum bit rate constraint.
4) As long as all the users meet the minimum bit rate constraint, redo step 1) until all the subcarriers have been allocated.

3.2. SM Scheme

In the SM scheme, the fact that each RT has \( Q \) receive antennas can be exploited in the spatial domain by transmitting up to \( Q \) independent bit streams simultaneously for each user. For specific \( n \), first choose among \( \{H_k^{n}\}_{k=1,2,\ldots,K} \) with the largest Frobenius norm
\[
k^* = \arg \max_{k \in \{1,2,\ldots,K\}} \|H_k^{n}\|_F.
\]
In the proposed SM scheme, if \( \rho_{k}^{\circ} = 1 \), then \( \rho_{k}^{\circ} = 0 \) for all \( k \neq k^* \).
In what follows, the output of the antenna array at the BTS can be described as
\[
x_k^{n} = \sum_{q=1}^{Q} w_{k,q}^{n} s_{k,q}^{n}
\]
where \( \{w_{k,q}^{n}\}_{q=1,2,\ldots,Q} \) is the weight vector designated for the information bits \( \{s_{k,q}^{n}\}_{q=1,2,\ldots,Q} \). The received \( Q \)-by-1 vector signal at the \( k^* \)th RT can be written as
\[ y_k^n = H_k^n x_k^n + n_k^n \]  \tag{23}

where \( n_k^n \) is the white noise vector with the covariance matrix \((N_c/2) I_Q\). For each subcarrier, the aim is transmitting \( Q \) bits simultaneously to the \( k \)-th user. The design goal of \( \{ w_{k,q}^n \}_{q=1,2,\ldots,Q} \) is to avoid ISI. In order to find appropriate weight vectors, the singular value decomposition (SVD) theorem is firstly employed to decompose \( H_k^n \) as

\[ H_k^n = U_k^T \Sigma_k \bar{V}_k^H \]  \tag{24}

where \( U_k^n \in \mathbb{C}^{Q \times Q} \) and \( \bar{V}_k^n \in \mathbb{C}^{L \times Q} \) are unitary matrices, and \( \Sigma_k^n \in \mathbb{C}^{Q \times Q} \) is a zero matrix except for the square roots of \( Q \) nonzero eigenvalues \((L>Q)\) of the matrix \( H_k^n H_k^n^H \) on the diagonals. Denote each diagonal by \( \{ \sqrt{\sigma_{k,q}} \}_{q=1,2,\ldots,Q} \). By the SVD theorem, the first \( Q \) columns of \( V_k^n \) are the orthonormal basis of the column space of \( H_k^n \), and therefore can be selected to be user \( k \)'s weight vectors.

\[ w_{k,q}^n = \eta_{k,q}^n \bar{v}_{k,q}; \quad q = 1, 2, \ldots, Q \]  \tag{25}

where \( \bar{v}_{k,q} \) denotes the \( q \)-th column vectors of \( V_k^n \). Based on (22) and (25), the power consumption of the SM scheme can be calculated as

\[ P_{SM} = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,n} E \left[ |s_{k,q}|^2 \right] \| w_{k,q}^n \|^2 = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,n} (\eta_{k,q}^n)^2. \]  \tag{26}

Substituting (22) and (24) to (25) into (23), we have

\[ H_k^n x_k^n = \left( U_k^T \Sigma_k \bar{V}_k^H \right) x_k^n = U_k^n \begin{bmatrix} \eta_{k,1}\sqrt{\sigma_{k,1}^n} s_{k,1}^n \\ \eta_{k,2}\sqrt{\sigma_{k,2}^n} s_{k,2}^n \\ \vdots \\ \eta_{k,Q}\sqrt{\sigma_{k,Q}^n} s_{k,Q}^n \end{bmatrix}. \]  \tag{27}

The optimum receiver of user \( k \) can be designed by matched-filtering the received signal with \( U_k^n \), which yields

\[ U_k^n y_k^n = U_k^n \left( H_k^n x_k^n + n_k^n \right) = U_k^n H_k^n x_k^n + n_k^n + \bar{U}_k^n \bar{V}_k x_k^n + \overline{\bar{U}_k^n \bar{V}_k x_k^n}. \]  \tag{28}

It is easy to show that \( \overline{\bar{U}_k^n \bar{V}_k x_k^n} = U_k^n n_k^n \) is still white with the same covariance matrix. It is evident from (28) that the \( Q \) bits, \( \{ s_{k,q}^n \}_{q=1,2,\ldots,Q} \), can be simultaneously detected. Substituting the bit energy \( E_{k,q}^n = E \left[ |n_{k,q}^n| \right] = \sigma_{k,q}^2 \rho_{k,n} \eta_{k,q}^n \) into (6), we have

\[ \eta_{k,q}^n \geq \sqrt{\frac{N_c}{2 \sigma_{k,q}^2}} Q^{-1} (\lambda). \]  \tag{29}

Substituting (29) into (26), the lower bound of the transmit power is obtained as

\[ P_{SM} \geq \frac{N_c}{2} Q^{-1} (\lambda) \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \frac{1}{\sigma_{k,q}^2}. \]  \tag{30}

The throughput of the \( k \)-th user yields

\[ T_k = (1 - \lambda) Q \sum_{n=1}^{N} \rho_{k,n} \eta_{k,q}^n; \quad k = 1, 2, \ldots, K. \]  \tag{31}
And the overall throughput is \( T = \sum_{k=1}^{K} T_k \). Consequently, the utility function of the SM scheme is

\[
U_{\text{SM}} \equiv \frac{T}{P_{\text{SM}}} \leq \frac{(1-\lambda) Q}{2 N_k (Q^{-1}(\lambda))^2} \sum_{k=1}^{K} \sum_{n=1}^{N_k} \rho_k^n + \frac{1}{\sigma_{k,q}^2}
\]

(32)

The power-efficient subcarriers allocation for the TM scheme is equivalent to finding \( \{\rho_k^n\}_{k=1}^{K} \) to satisfy the following constrained-optimization problem \( \arg \max_{\rho_k^n} U_{\text{SM}} \).

Subject to:
1) \( R_k = Q \sum_{n=1}^{N_k} \rho_k^n \geq R_{k,\text{min}}; \quad k = 1, 2, \ldots, K \). \hspace{1cm} (33)

2) If \( \rho_k^n = 1 \Rightarrow \rho_{k'}^n = 0, \forall k' \neq k \), hence

\[
\sum_{k=1}^{K} \sum_{n=1}^{N_k} \rho_k^n = N.
\]

(34)

The algorithm of the proposed SM scheme can be summarized as follows.

1) Starting from the first subcarrier, assign the subcarrier to the user with the highest Frobenius norm of the channel matrix in that subchannel. Let \( W \) be the user set, \( W = \{1, 2, \ldots, K\} \), then

\[
k^* = \arg \max_{k \in W} \| H_k^o \|_F.
\]

As soon as subcarrier \( n \) has been assigned to user \( k^* \), set \( \rho_{k^n}^n = 1, \rho_{k'}^n = 0 \), for all \( k \neq k^* \).

2) Perform SVD on \( H_k^o \) to obtain \( \{ \psi_k^o \}_{q=1}^{Q} \).

3) Calculate the cumulative transmission rate

\[
R_{k'} = Q \sum_{n=1}^{N_k} \rho_{k'}^n.
\]

(35)

4) Check whether \( R_{k'} \) meets the minimum bit rate constraint. If \( R_{k'} < R_{k',\text{min}} \), which means more subchannels should be assigned to user \( k' \), go back to step 1) and check for the next subcarrier. On the other hand, if \( R_{k'} \geq R_{k',\text{min}} \), temporarily exclude user \( k^* \) from the set of \( W \),

\[
W - \{k^*, q\} = \{(k, q)\}_{k=1}^{K} \setminus \{k^*, q\}, q=1, 2, \ldots, Q, k^*, q^* \}
\]

and go back to step 1) to assign the subchannels to those users that have not achieved the minimum bit rate strain.

5) As long as all the users meet the minimum bit rate constraint, redo step 1) until all the subcarriers have been allocated.

3.3. SDMA Scheme

Different from the above two schemes, a subcarrier can be assigned simultaneously to multiple users in the SDMA scheme. In the proposed SDMA scheme, the “most orthogonal” user terminals are systematically assigned to the same subchannel, hence, the overall throughput is expected to increase. It is plausible to measure the orthogonality of two user terminals at subcarrier \( n \) by the crosscorrelation magnitude between their spatial signatures, \( |h_{k,n}^o h_{k',q}^o| \) for \( \forall k \neq k', q \neq q' \).
Let $W_n$ be the user set with spatial signatures for the $n$th subcarrier, which are pairwise approximately orthogonal, then

$$h_{k,q}^n h_{k',q'}^n = 0; \forall (k, q), (k', q') \in W_n. \quad (36)$$

Since the users in $W_n$ are allowed to transmit simultaneously on subcarrier $n$ without causing interference at the receiving end, the subcarrier allocation index for the $n$th subcarrier yields

$$\rho_{k,q}^n = \begin{cases} 1; & (k,q) \in W_n \\ 0; & \text{otherwise}. \end{cases} \quad (37)$$

Therefore, the transmitted signal on subcarrier $n$ can be written by

$$x^n = \sum_{(k,q)\in W_n} w_{k,q}^n s_{k,q}^n. \quad (38)$$

Since the channel vectors are pairwise orthogonal, the weight vector for transmit beamforming can be designed as

$$w_{k,q}^n = \eta_{k,q}^n h_{k,q}^n; \quad (k,q) \in W_n. \quad (39)$$

Substituting (39) into (38), the power consumption can be obtained as

$$P_{SDMA} = \sum_{n=1}^N \sum_{(k, q) \in W_n} \rho_{k,q}^n \mathbb{E} \left[ |s_{k,q}^n|^2 \right] \|w_{k,q}^n\|^2 = \sum_{n=1}^N \sum_{(k, q) \in W_n} \rho_{k,q}^n \eta_{k,q}^n \|h_{k,q}^n\|^2. \quad (40)$$

For specific $(k, q) \in W_n$, the received signal at the $k$th user’s $q$th antenna can be written by

$$y_{k,q}^n = h_{k,q}^n \left( \sum_{(k', q') \in W_n} w_{k',q'}^n s_{k',q'}^n h_{k',q'}^n \right) + n_{k,q}^n = \eta_{k,q}^n s_{k,q}^n \|h_{k,q}^n\|^2 + h_{k,q}^n \left( \sum_{(k', q') \in W_n} \eta_{k',q'}^n s_{k',q'}^n h_{k',q'}^n \right) + n_{k,q}^n \approx \eta_{k,q}^n s_{k,q}^n \|h_{k,q}^n\|^2 + n_{k,q}^n \quad (41)$$

where $(k, q) \in W_n$.

Please note that in (41), the residual MUI has been neglected based on the fact of (36). In view of (11) and (41), the received signal model is the same as the TSD scheme, therefore, the following results are straightforward:

$$\eta_{k,q}^n \geq \frac{Q^{-1}(\lambda)}{\|h_{k,q}^n\|^2} \sqrt{\frac{N_0}{2}} \quad (42)$$

$$P_{SDMA} \geq \frac{N_0}{2} \left( Q^{-1}(\lambda) \right)^2 \sum_{n=1}^N \sum_{(k,q)\in W_n} \rho_{k,q}^n \frac{1}{\|h_{k,q}^n\|^2}. \quad (43)$$

The throughput of each user yields

$$T_k = (1 - \lambda) \sum_{n=1}^N \sum_{(k,q)\in W_n} \rho_{k,q}^n; \quad k = 1, 2, \ldots, K. \quad (44)$$

Using (43) and (44), the utility function can be obtained as

$$U_{SDMA} \equiv \frac{T}{P_{SDMA}} \leq \frac{(1 - \lambda) \sum_{n=1}^N \sum_{(k,q)\in W_n} \rho_{k,q}^n}{\frac{N_0}{2} \left( Q^{-1}(\lambda) \right)^2 \sum_{n=1}^N \sum_{(k,q)\in W_n} \rho_{k,q}^n \frac{1}{\|h_{k,q}^n\|^2}}. \quad (45)$$

The algorithm of the proposed SDMA scheme is proceeded in the following.
1) Grouping: Starting from the first subcarrier, BTS first separates and groups all the spatial signatures \( \{ h_{k,q}^n \}_{q=1,2,\ldots,Q} \) into a set of quasi-orthogonal subsets based on a predetermined criterion

\[
| h_{k,q}^n h_{k',q'}^n | \leq \alpha; \forall (k,q) \neq (k',q') \tag{46}
\]

where \( \alpha \) is a small positive value. We denote the quasi-orthogonal subset for the \( n \)th subcarrier as \( \{ W_i^n \}_{i=1,2,\ldots} \).

2) Selecting: Denoting the sum norm of each vector in \( W_i^n \) as \( \| W_i^n \| \), BTS assigns the \( n \)th subcarrier to all the users in \( W_i^n \), which yields the highest sum norm

\[
i^* = \arg \max_i \| W_i^n \| . \tag{47}
\]

And immediately set the subcarrier allocation index for the \( n \)th subcarrier as

\[
\rho_{k,q}^n = \begin{cases} 1: \ (k,q) \in W_i^n \\ 0: \ \text{otherwise.} \end{cases}
\]

3) Steps 1) and 2) are proceeded until all the subcarriers have been allocated, i.e., \( \{ W_i^n \}_{i=1,2,\ldots,N} \) \( \{ \rho_{k,q}^n \} \) is created and the number of transmitted bits per unit time can be calculated by

\[
N = \sum_{n=1}^N \sum_{(k,q) \in W_i^n} \rho_{k,q}^n. \tag{48}
\]

4) Calculate the transmission rate

\[
R_k = \sum_{n=1}^N \sum_{(k,q) \in W_i^n} \rho_{k,q}^n; \quad k = 1, 2, \ldots, K .
\]

5) Reallocating: Starting from the first user, check whether the minimum bit rate constraint is attained. If \( R_k \geq R_{k,\min} \), check for the next user, otherwise, increase the threshold value from \( \alpha \) to \( \alpha + \Delta \), where \( \Delta \) is a small positive value, redo steps 2) to 5), until \( R_k \geq R_{k,\min}; \quad k = 1, 2, \ldots, K \).

3.4. ZF Scheme

The weight vector of the ZF scheme is selected based on the ZF criterion such that each user’s transmission does not interfere with other users’ data. Consequently, MUI is completely removed by transmit beamforming and each subcarrier can be assigned simultaneously to all the user terminals. To mitigate MUI, the weight vector should meet the following criterion

\[
h_{k,q}^n w_{k,q}^n = \eta_{k,q}^n \delta(k-k',q-q') = \begin{cases} \eta_{k,q}^n; \ k = k', \ q = q' \\ 0; \ \text{otherwise.} \end{cases} \tag{49}
\]

Upon defining the \( L \)-by-\( KQ \) matrix, \( H^n \equiv \left[ H_1^n \quad H_2^n \quad \cdots \quad H_K^n \right] \), (49) can be rewritten as a compact form

\[
H^n w_{k,q}^n = \eta_{k,q}^n e_{(k-1)Q+q}. \tag{50}
\]

If the array size of BTS satisfies \( L \geq KQ \), which is usually the case in the massive MIMO system\(^{[14][15]}\), there are infinitely many solutions since (50) is an underdetermined system. Assuming that \( H^n \) is full column rank, then the minimum-norm solution of (50) yields

\[
w_{k,q}^n = \eta_{k,q}^n H^n (H^n H^n)^{-1} e_{(k-1)Q+q}. \tag{51}
\]

Since for arbitrary \( n \), all user terminals are allowed to share a subchannel at the same time, hence, the subcarrier allocation index for the ZF scheme is
\[ \rho_{k,q}^n = 1; \ \forall k = 1, 2, \ldots, K; n = 1, 2, \ldots, N; q = 1, 2, \ldots, Q. \]  \tag{52} 

The power consumption of the ZF scheme can be calculated from (51) and (52) as

\[
P_{ZF} = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n \| w_{k,q}^n \|^2
= \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \eta_{k,q}^n \| H_n^T [H_n^T H_n]^{-1} e_{(k-1)Q+q} \|^2
= \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \eta_{k,q}^n \| H_n^T [H_n^T H_n]^{-1} (k-1) Q+q, (k-1) Q+q \|. \tag{53} \]

The received signal at the \( k \)th RT's \( q \)th receiver is

\[ y_{k,q}^n = h_{k,q}^n [\sum_{k=1}^{K} \sum_{n=1}^{N} s_{k,q} [w_{k,q}^n]^T ] + n_{k,q}^n = \eta_{k,q}^n s_{k,q} + n_{k,q}^n. \tag{54} \]

As revealed by (54), though multiple users are simultaneously transmitted using the same subcarrier, only the desired signal remains, in which the received bit energy is \( E_{k,q}^{ZF} = E \{ |\eta_{k,q}^n s_{k,q}^2 |^2 \} = \eta_{k,q}^n \). To maintain the required QoS, we have

\[ \eta_{k,q}^n \geq Q^{-1} (\lambda) \sqrt{\frac{N_0}{2}}. \] \tag{55} 

Therefore,

\[
P_{ZF} \geq \frac{N_0}{2} [Q^{-1} (\lambda)]^2 \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \| H_n^T [H_n^T H_n]^{-1} (k-1) Q+q, (k-1) Q+q \| = \frac{N_0}{2} [Q^{-1} (\lambda)]^2 \sum_{n=1}^{N} \text{tr} \left( [H_n^T H_n]^{-1} \right). \tag{56} \]

The utility function can be obtained as

\[
U_{ZF} \equiv \frac{T}{P_{ZF}} \leq \frac{(1-\lambda) \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{q=1}^{Q} \rho_{k,q}^n}{\frac{N_0}{2} [Q^{-1} (\lambda)]^2 \sum_{n=1}^{N} \text{tr} \left( [H_n^T H_n]^{-1} \right)} = \frac{(1-\lambda) N Q K}{\frac{N_0}{2} [Q^{-1} (\lambda)]^2 \sum_{n=1}^{N} \text{tr} \left( [H_n^T H_n]^{-1} \right)}. \tag{57} \]

If, on the other hand, \( L<KQ \), it is not possible to use the ZF scheme since the matrix \( H_n^T H_n \) in (51) is now singular. To make the ZF scheme feasible, one needs to select \( L \) out of the \( KQ \) column vectors of \( H_n^T \). It is called as the SZF scheme. The objective of the SZF scheme is to allocate each subcarrier to \( L \) spatial signatures selected from \( H_n^T \).

4. Performance Evaluation

In this section, the performance of the proposed resource allocation schemes is evaluated and compared. Unless otherwise mentioned, set the target QoS (BER upper bound) to be \( \lambda = 10^{-3} \) for the simulation examples. For each subcarrier, generate the channel vector \( h_{k,q}^n = \sqrt{\alpha_{k,q}^n} g_{k,q}^n \), in which \( g_{k,q}^n \) denotes small-scale fading and \( \alpha_{k,q}^n \) denotes the large-scale fading coefficient (lognormal distribution and geometric decay). Assume \( g_{k,q}^n \sim \mathcal{CN}(0, \text{I}) \), statistically independent across users. The result in each simulation example is obtained from the average of 100 independent trials. The predetermined threshold, \( \alpha \), and step size, \( \Delta \), for the SDMA scheme is set to be 1 and 0.5, respectively. To treat users fairly, set the equal rate constraint on each user, i.e., \( R_{1,\text{min}} = R_{2,\text{min}} = \cdots = R_{k,\text{min}} = R \) bits per unit time. Note that in each simulation example, the ZF and SZF schemes are applied interchangeably in the cases of \( L>KQ \) and \( L<KQ \), respectively.
Fig. 2 presents the energy efficiency or utility with respect to the number of user terminals (ranging from $K=4$ to $K=40$), where $N=300$, and the array size at BTS and each RT are $L=60$ and $Q=3$, respectively. As shown in Fig. 2, the utility is in an ascending order from the ZF (SZF) to SM schemes, then the SDMA scheme, and the TSD scheme is the best. Somewhat surprisingly, the utility of the ZF scheme is the worst (lowest) among all the proposed schemes, even though its throughput is the highest since all user terminals are allowed to share a subcarrier at the same time. This is due to the fact that more power is consumed in order to complete the MUI mitigation. On the other hand, though only one of $KQ$ channels is active for information transmission for the TSD scheme, it enjoys the highest utility since the power consumption is extremely low. Specifically, the utility of the SDMA scheme increases for larger $K$. This may result from the fact that as $K$ increases, more orthogonal channels can be selected, which allows multiple user terminals to share a subchannel at the same time. Thereby, larger $K$ increases the throughput for the SDMA scheme.

Fig. 3 presents the energy efficiency with respect to the number of the BTS antennas (varying from $L=20$ to $L=200$), where $N=256$, $K=20$, and $Q=5$. As depicted in Fig. 3, the utility of all the proposed schemes increases in accordance with larger $L$. The TSD scheme is the most sensitive to $L$ since the increase of the diversity order ensures that the subcarrier can be allocated to the user with a better channel condition. Moreover, when $L$ grows largely, the random channel vectors between the users and BTS become pairwisely orthogonal\cite{11}, which evidently improves the system performance. Fig. 4 compares the energy efficiency with respect to QoS (BER ranges from $\lambda = 10^{-4}$ to $\lambda = 0.0181$). Set the parameters as $N=256$, $K=20$, $Q=5$, and $L=60$. As expected, as $\lambda$ increases, the averaged power required to successfully transmit a bit is decreased. The aim of the final simulation example is to evaluate the performance improvement invoked by the spatial diversity at RT. Present the energy efficiency with respect to the number of RT’s antennas (ranging from $Q=1$ to $Q=19$) in Fig. 5, where $N=300$, $K=20$, and $L=200$, respectively. As revealed by Fig. 5, both the ZF and SM schemes are insensitive to the increase of $Q$. The reason is similar to the above discussion that
increasing the spatial multiplexing at RT leads to more power consumption.

As a whole, according to the simulation results, the following observations can be drawn:

1) Though the ZF and SM schemes have an excellent throughput performance, nevertheless, the energy efficiency is quite low. The main reason is large power consumption used to separate the multiple users' transmission. On the other hand, the energy efficiency of the TSD scheme is the best, whereas it suffers from poor throughput since it is inherently a single-user resource allocation algorithm.

2) As verified by the simulation results, the SDMA scheme can be regarded as a tradeoff between the system throughput and energy efficiency. In SDMA, the most orthogonal spatial signatures are chosen to share the same subcarrier simultaneously, hence, both the throughput and energy efficiency performances are improved.

5. Conclusions

In this paper, the throughput is extensively analyzed, as well as the power consumption of four resource allocation algorithms in the OFDMA-based MIMO system, namely, the TSD, SM, SDMA, and ZF (SZF) schemes. Moreover, a utility function is exploited to measure the energy efficiency of the four schemes. Analytical and simulation results demonstrate that the TSD scheme has the best energy efficiency. It has been also verified that employing the antenna array at BTS and/or RT has extensively increased the energy efficiency of the TSD and SDMA schemes. Specifically, the SDMA algorithm, which judiciously selects co-channel users based on their orthogonality between their spatial signatures, is found to enable a significant improvement in the system throughput under low power consumption. This paper has provided a comprehensive study of the multi-user resource allocation in 5G massive MIMO and green radio networks.

References


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