Energy Harvesting in Cognitive Networks with Cooperative Beamforming: Power Allocation and Stability Analysis

Jin-Xin Niu* | Wei Guo

Abstract—Energy harvesting (EH) is a promising technology to improve both energy efficiency and spectral efficiency in cognitive radio (CR) networks. However, due to the randomness of the harvested energy and the interference constraint at the primary users (PUs), the limited transmission power of secondary users (SUs) may reduce the service rate of SUs. To solve this problem, this paper investigates a cooperative transmission method where a zero-forcing beamforming method is used in the EH based secondary network. Considering the transmission power constraint and energy causality, we derive the closed-form solution of the optimal transmission power for the secondary source and relays, which achieves the maximal stable throughput of the secondary network. Numerical results show the impact of different system parameters to the maximal stable throughput. In addition, compared with the traditional decode-and-forward (DF) scheme, the cooperative beamforming method achieves higher stable throughput under an high quality source-to-relay channel.

Index Terms—Cooperative beamforming, energy harvesting, power allocation, stability analysis.

1. Introduction

Energy harvesting (EH) enables nodes to harvest energy from nature environment through solar cells, water mills, etc., to maximize network lifetime or save energy. It has been considered as a promising technology in green communications networks[1, 2].

In recent years, research has been focused on integrating EH technology into existing scenarios of wireless communications networks. Due to the capability of improving both energy efficiency and spectral efficiency, EH based cognitive radio (CR) networks have been investigated recently[3, 4]. In [3], the achievable throughput of the secondary transmitter was derived as a function of the energy arrival rate, primary traffic, and detection threshold of a spectrum sensor. In [4], the outage performance of the secondary network was investigated under the harvested radio frequency (RF) energy constraint of secondary users (SUs) and the interference constraint of the primary

*Corresponding author
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J.-X. Niu and W. Guo are with the National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: njx_666@126.com; guowei@uestc.edu.cn).
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users (PUs). Reference [5] proposed an optimal spectrum sensing policy which maximized the total throughput under the constraint of energy causality and collision.

To further improve the secondary network performance, some researchers considered the potential of integrating the cooperative communications technology into the EH based CR networks [6]-[11]. In these works, [9] and [10] considered EH based underlay cognitive relay networks. The derived outage probability expression of SUs in [9] was subject to the harvested energy and maximal transmission power. While in [10], the outage performance of SU was analyzed by considering the harvested energy and the interference from PUs. In [11], PU exploited the SU power to relay packets, and SU obtained more opportunities to access idle time slots. However, the main drawback of these works is that SUs can only access idle spectrum or adjust the transmission power of SUs when PUs are active, which reduces the service rate of SUs. Considering the randomness of energy arrival process, the performance of the secondary network may be further degraded.

In this paper, in order to improve the service rate of the EH enabled SUs, the EH based cognitive radio scenario with cooperative beamforming is approached. The secondary source node collects energy from the surrounding environment, and the relays are connected to a centralized energy harvester [12]. The EH secondary source and relays adopt both power control and the zero-forcing beamforming method to share the spectrum with PU, which increases the service rate of SUs. Furthermore, in order to achieve the maximal stable system throughput, the closed-form expressions of the optimal transmission power of the secondary source and relays are derived by considering the randomness of the energy packets and the power constraint. The impact of different system parameters to the maximal stable throughput is shown by using the numerical results. In addition, the system performance is compared with the conventional decode-and-forward (DF) system. It shows that the cooperative beamforming system with the proposed power allocation method can achieve higher stable throughput than the DF based method under an high quality source-to-relay channel. To our best knowledge, the stability analysis of the EH based cognitive networks with cooperative beamforming is first time studied in this paper.

2. System Model

We consider a simple CR system consisting of a PU, a secondary source s, and a secondary destination d. There is no direct link between s and d. The source s communicates with d via a set of relay nodes R which contains K relays. The source s harvests energy from the surrounding environment. The relays in R are connected to a centralized energy harvester, which distributes the harvested energy to the relays using cables [12]. The transmission power of s is $P_s$. The total transmission power of the relays in R is $P_R$. To prevent interference to PU, $P_s$ is kept below a threshold in the first hop transmission of SU. The zero-forcing beamforming method [13] which causes no interference to PU is adopted in the second hop transmission.

For the sources, there are a data packet queue and an energy packet queue. For the relays in R, each relay has a data queue. And all the relays are connected to a centralized energy harvester which is equipped with an energy packet queue. All queues are assumed to have an infinite capacity [14]. The arrival models of data packets and energy packets follow the Poisson process with parameters $\lambda_d$ and $\lambda_e$, respectively. A time-normalized slotted packet system is considered, and the transmission of each data packet requires one slot.

Each data packet contains $R_{th}$ bits that correspond to a spectral efficiency of $R_{th}$ bits per channel use (BPCU) [15]. In the first hop transmission, a data packet can be successfully decoded by a relay node r in R when $\log_2(1 + P_s|h_{sr}|^2/P_T) \geq R_{th}$, where $P_T$ is the sum of the interference from PU and the additive white Gaussian noise (AWGN), and $h_{sr}$ is the Rayleigh fading channel coefficients between s and r. If a data packet is successfully received by at least K relays, the data packet is removed from the data queue of s. Otherwise, this data packet will...
be retransmitted. Without loss of generality, we consider that $K_{th} = K$. In this case, the probability that one data packet from $s$ is successfully received by $K_{th}$ relays can be expressed by

$$
\Pr_{success}^{sR} = \frac{\Pr \left[ \log_2 (1 + P_s |h_{sr}|^2 / P_R) \geq R_{th} \right]^K}{\Pr \left( P_s \right)}
$$

where $\Pr(P_s)$ is the probability that the power $P_s$ is available at the energy queue of node $s$ and is given as

$$
\Pr(P_s) = \begin{cases} 
2 \lambda_s / P_s, & P_s > 2 \lambda_s \\
1, & \text{otherwise.}
\end{cases}
$$

In (2), the factor 2 implies that transmitting one data packet from $s$ to $d$ needs two consecutive slots: The first slot is for the transmission from $s$ to $R$, and the second one is for the transmission from $R$ to $d$. In addition, $P_s$ should be limited to protect PU, thus we have $P_s \leq P_{th}$.

In the second hop transmission, all relay nodes simultaneously transmit the decoded packet to the destination using the zero-forcing beamforming method. Since the relays in $R$ simultaneously transmit and receive data packets, they have the same data packet arrival and service processes. The successful transmission probability of the second hop is

$$
\Pr_{success}^{Rd} = \Pr \left[ \log_2 (1 + |h|^2 g^2 P_R / P_T) \geq R_{th} \right] \Pr(P_R)
$$

where $h$ is the Rayleigh fading channel vector composed of the channel coefficient $h_{rd}$ between the relay $r$ and the destination $d$, and $g$ is the beamforming vector of the relays. For the zero-forcing beamforming method, $g$ is the orthogonal projection of $h$ onto the orthogonal complementary of the subspace $\text{Span}\{h_1\}$, where $h_1$ is the channel coefficient vector between relays and the primary receiver, and the random variable $|h|^2 g^2$ obeys the Chi-square distribution with $2L$ ($L = K - 1$) degrees of freedom. Therefore, (3) can be rewritten as

$$
\Pr_{success}^{Rd} = \Pr(P_R) \sum_{l=0}^{L-1} \frac{1}{l!} A^l \exp(-A)
$$

where $A = (2^{R_{th}} - 1)P_R / P_T$, and $\Pr(P_R)$ denotes the probability that power $P_R$ is available at the energy queue of the centralized energy harvester. Similar to (2), $\Pr(P_R)$ is given as

$$
\Pr(P_R) = \begin{cases} 
2 \lambda_R / P_R, & P_R > 2 \lambda_R \\
1, & \text{otherwise.}
\end{cases}
$$

3. Maximal Stable Throughput and Optimal Transmission Power

In this section, we derive the optimal transmission power of the secondary source and relays which achieve the maximal stable throughput of the secondary network. Firstly, the conditions of a stable secondary network are analyzed. Then, the optimal total transmission power of the relays is derived. Lastly, according to the maximal achieved stable throughput of the relays, the optimal transmission power of the secondary source is obtained.

3.1. Conditions of the Stable Secondary Network

The stability of the secondary network requires that the data queues of the second source and relays are all stable. However, in EH networks, nodes should wait until there is enough energy in the energy queue to transmit the data packets. This interaction between the data queue and energy queue complicates the stability analysis. We
exploit the dominant system concept\textsuperscript{[16]} to decouple the interaction. In a dominant system, the secondary source and relays transmit dummy packets even if their data queues are empty. Therefore, the secondary source and all relays continuously consume power $P_s$ and $P_r$, respectively. In this case, as transmitting one data packet from $s$ to $d$ needs two consecutive slots, the service rates of the source and relays data queues are $Pr_{\text{success}}/2$ and $Pr_{\text{rd success}}/2$, respectively.

For the data queue of the source node, the arrival rate of the data packets is $\lambda_d$. For the data queues of the relays, the arrival rate of data packets is the service rate of the source node. In the dominant system, the source node transmits its own packets or dummy packets, thus the data queue is always non-empty. Therefore, the arrival rate of the data queues of the relays is $Pr_{\text{rd success}}/2$.

According to the Loynes’ theorem\textsuperscript{[17]}, if the mean arrival rate of a queue is less than the mean service rate, the queue is stable, otherwise it is unstable. Based on the above discussion, the stability conditions of the secondary network are given as

$$\lambda_d < Pr_{\text{rd success}}/2$$  \hspace{1cm} (6)

$$Pr_{\text{rd success}}/2 < Pr_{\text{rd success}}/2$$  \hspace{1cm} (7)

$$P_s \leq P_{\text{th}}$$  \hspace{1cm} (8)

where (6) and (7) correspond to the data queues stability conditions of the source and relays, and (8) is the power constraint of the source which is to protect the PU.

In the dominant system, the data queues of the secondary source and relays are always non-empty, thus, the queue length in the dominant system is longer than that in the original system. It is inferred that if the data queues in the dominant system are stable, they are also stable in the original system. On the other hand, if the data queues in the dominant system are saturated, the secondary source and relays will always transmit their own data packets instead of dummy packets. Therefore, the dominant system and original system are indistinguishable at boundary points, and the maximal stable throughput of the original system can be obtained through the dominant system based method at the boundary case.

### 3.2. Optimal Total Transmission Power of the Relays

As mentioned in Section 2, in the zero-forcing beamforming method, we need to calculate the normalized beamforming vector $\mathbf{g}$ and the total transmission power $P_R$. And $\mathbf{g}$ is obtained in Section 2. So, we calculate $P_R$ in this subsection.

According to (6) and (7), to obtain the maximal stable system throughput $\max\{\lambda_d\}$, $Pr_{\text{rd success}}$ should be maximized first. Based on (4) and (5), when $P_R \leq 2\lambda_w$, we have $Pr(P_R) = 1$ and

$$Pr_{\text{rd success}} = \sum_{l=0}^{L-1} \frac{1}{l!} A^l \exp(-A)$$  \hspace{1cm} (9)

$$\frac{\partial Pr_{\text{rd success}}}{\partial P_R} = \exp(-A) \frac{(2P_{\text{th}} - 1)P_T}{P_R^2} \frac{1}{(L-1)!} A^{L-1} > 0. $$  \hspace{1cm} (10)

In this case, $Pr_{\text{rd success}}$ monotonically increases with $P_R$. Thus, we have $P_R = 2\lambda_w$.

When $P_R > 2\lambda_w$, we have $Pr(P_R) = 2\lambda_w/P_R$ and

$$Pr_{\text{rd success}} = \frac{2\lambda_w}{P_R} \sum_{l=0}^{L-1} \frac{1}{l!} A^l \exp(-A)$$  \hspace{1cm} (11)
The high signal to interference plus noise ratio (SINR) approximation of (12) is less than zero, and $P_{\text{Rd succ}}$ monotonically decreases as $P_R$. Thus, we have $P_R = 2\lambda_e$.

Based on (10) and (12), $P_{\text{Rd succ}}$ is maximized when $P_R = 2\lambda_e$.

### 3.3. Optimal Transmission Power of the Source

The optimal transmission power of the source $P_s$ can be obtained through maximizing $P_{\text{Rd succ}}$ under the constraints (7) and (8). The maximum of $P_{\text{Rd succ}}$ has been derived in subsection 3.2. Assume that $\max \{P_{\text{Rd succ}}\} = \psi$. Therefore, (7) can be re-written as

$$P_{\text{Rd succ}} < \psi. \quad (13)$$

Considering the expression of $P_{\text{Rd succ}}$ in (1), and the constraints (8) and (13), we can obtain the optimal transmission power of the source $P_s$ as the following proposition.

**Proposition 1.** Assume that $M = K(2Rth - 1)P_t$, $N = \exp(-1)2\lambda_e/M$, $J = \exp(-M/2\lambda_e)$, $\alpha = \lambda_e[1 - \sqrt{1 - 2\psi M/\lambda_e}] / \psi$, and $\beta = \lambda_e[1 + \sqrt{1 - 2\psi M/\lambda_e}] / \psi$. The optimal transmission power of the source which enables the maximal stable throughput of the secondary network can be described as follows:

- If $(P_{\text{th}} > 2\lambda_e$ and $N < \psi$ and $M \leq 2\lambda_e$), or $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $M \leq 2\lambda_e$ and $J \leq \psi$), then $P_s = 2\lambda_e$;
- If $(P_{\text{th}} > 2\lambda_e$ and $N < \psi$ and $2\lambda_e < P_{\text{th}} \leq M$), or $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $M \geq P_{\text{th}}$ and $J \leq \psi$), then $P_s = P_{\text{th}}$;
- If $(P_{\text{th}} > 2\lambda_e$ and $N < \psi$ and $2\lambda_e < M < P_{\text{th}}$), then $P_s = M$;
- If $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $2\lambda_e < M < P_{\text{th}}$ and $2\lambda_e \leq \alpha < \min(\beta, P_{\text{th}}))$, or $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $M \geq P_{\text{th}}$ and $J > \psi$), then $P_s = \alpha$;
- If $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $M \leq 2\lambda_e$ and $J > \psi$), or $(P_{\text{th}} > 2\lambda_e$ and $N \geq \psi$ and $2\lambda_e < M < P_{\text{th}}$ and $\max(\alpha, 2\lambda_e) < \beta \leq P_{\text{th}})$, then $P_s = \beta$;
- Otherwise, $P_s = \min(M / \ln \psi, P_{\text{th}}, 2\lambda_e)$.

*Proof. The proof is shown in the appendix section.

### 4. Numerical Results

In this section, the maximal stable throughput of the secondary networks is simulated with different system parameters. The number of relays $K = 4$.

Fig. 1 depicts the maximal stable throughput with different energy packets arrival rates $\lambda_e$ and the given decode threshold $R_{\text{th}}$ when $P_t = 1$ energy packets and $P_{\text{th}} = 20$ energy packets. The maximal stable throughput increases as $\lambda_e$ increases until the curve becomes flat at $\lambda_e = 10$ energy packets/slot. The reason is that a larger $\lambda_e$ means that the nodes can transmit data packets with higher power. When enough energy is harvested, the transmission power of the source node is only limited by $P_{\text{th}}$. The maximal stable throughput increases as $\lambda_e$ increases until the curve becomes flat at $\lambda_e = 10$ energy packets/slot. The reason is that a larger $\lambda_e$ means that the nodes can transmit data packets with higher power. When enough energy is harvested, the transmission power of the source node is only limited by $P_{\text{th}}$. The maximal stable throughput.

![Maximal stable throughput versus different $\lambda_e$ and $R_{\text{th}}$.](image-url)
throughput increases as $R_{th}$ decreases, since a small $R_{th}$ implies the high probability of successful transmission, leading to higher stable throughput.

Fig. 2 illustrates the maximal throughput with different $\lambda_e$ and interference powers $P_T$ when $R_{th} = 1$ BPCU and $P_{th} = 20$ energy packets. A small $P_T$ value leads to the high probability of successful transmission and enhances the stable throughput. Therefore, the system stable decreases as $P_T$ increases.

Fig. 3 compares the stable system throughput with different $\lambda_e$ and $P_{th}$, when $R_{th} = 1$ BPCU and $P_T = 1$ energy packets. The three curves overlap together when $\lambda_e < 5$ energy packets/slot. The reason is that when $\lambda_e$ is small, the source node has less available energy, thus, the transmission power cannot reach the power limitation $P_{th}$, and the maximal stable throughput increases as $\lambda_e$ increases. The curve for $P_{th} = 10$ energy packets becomes flat first when $\lambda_e = 5$ energy packets/slot. The reason is that although the source can obtain more energy when $\lambda_e$ becomes larger, the transmission power is still constrained by $P_{th}$, limiting the service rate of the first hop transmission. The curve for $P_{th} = 20$ becomes flat last, because a larger $P_{th}$ increases the service rate of the first hop transmission, and thus the system obtains higher system throughput.

In Figs. 4 and 5, we compare the performance between the ordinary DF scheme and the EH based cooperative beamforming method in the CR system. In the DF system, the transmission power of the source and relay are limited by $P_{th}$. Fig. 4 shows that the DF scheme can obtain higher stable system throughput than the cooperative beamforming method. The reason is that there is only one relay node in DF system, and the probability of the successful first hop transmission in the DF system is higher than that in the cooperative beamforming based system, where the successful first hop transmission requires $K$ relays decode the data packet from the source. However, in practical scenario, the relays are usually selected from proximate neighbors, and the probability of successful first hop transmission may be greater. As shown in Fig. 5, if we relax the first hop constraint and assume that the first hop transmission is perfect as that in [14], the stable throughput of cooperative beamforming system is higher than that of the DF scheme. As discussed in Section 3, the optimal total transmission power of the relay set is $2\lambda_e$. Thus, the
The service rate of cooperative beamforming based system increases as $\lambda_e$ increases. In the DF system, the stable throughput remains constant when $\lambda_e \geq 10$ energy packets/slot, since the transmission power of the relay node is limited to protect the PU, and the service rate of the relays is reduced.

5. Conclusion

This paper analyzed the stability problem in EH cognitive networks with cooperative beamforming. The maximal stable throughput and the corresponding optimal transmission power of the secondary source and relays were obtained in closed forms. Furthermore, the maximal stable throughput of the secondary network with different system parameters was analyzed and discussed. Simulation results showed that the zero-forcing beamforming method achieves higher stable throughput than the ordinary DF scheme under the assumption of the perfect first hop transmission.

Appendix

The Proof of Proposition 1

The maximum of $Pr_{\text{success}}^{\text{BF}}$ should be derived under two constraints: $Pr_{\text{success}}^{\text{BF}} < \max\{Pr_{\text{success}}^{\text{DF}}\}$ and $P_s \leq P_{\text{th}}$. Recall that $\max\{Pr_{\text{success}}^{\text{DF}}\} = \psi$, $K(2^{R_{\text{th}}} - 1)P_T = M$, $\exp(-1)2\lambda T/M = N$, $\exp[-M/2\lambda T] = J$, $a = \lambda T \left[1 - \sqrt{1 - 2\mu M/\lambda T}\right]/\psi$, and $\beta = \lambda T \left[1 + \sqrt{1 - 2\mu M/\lambda T}\right]/\psi$.

If $Pr(P_s) = 1(P_s \leq 2\lambda_e)$, $Pr_{\text{success}}^{\text{BF}} < \psi \Rightarrow P_s < -M/ln \psi$. Considering $P_s \leq P_{\text{th}}$ and $P_s \leq 2\lambda_e$, so in this case

$$P_s = \min[-M/ln \psi, P_{\text{th}}, 2\lambda_e].$$

If $Pr(P_s) = 2\lambda_e/P_s (P_s > 2\lambda_e)$, $Pr_{\text{success}}^{\text{BF}} = 2\lambda_e \exp[-M/P_s]/P_s$, then $\partial Pr_{\text{success}}^{\text{BF}}/\partial P_s = 0 \Rightarrow P_s = M$.

So when $P_s = M$, $Pr_{\text{success}}^{\text{DF}} = N$. Considering $P_s \leq P_{\text{th}}$, $P_s > 2\lambda_e$, and $Pr_{\text{success}}^{\text{BF}} < \psi$, the maximum of $Pr_{\text{success}}^{\text{DF}}$ should be discussed as the following cases:

(C1) If $N < \psi$,

(C1.1) If $P_{\text{th}} \geq 2\lambda_e$,

(C1.1.1) If $M \leq 2\lambda_e$, $P_s = 2\lambda_e$;

(C1.1.2) If $2\lambda_e < M < P_{\text{th}}$,

then $P_s = M$;

(C1.1.3) If $2\lambda_e < P_{\text{th}} \leq M$,

then $P_s = P_{\text{th}}$.

(C2) If $N \geq \psi$,

(C2.1) If $P_{\text{th}} \geq 2\lambda_e$,

In this situation, the maximum of $Pr_{\text{success}}^{\text{DF}}$ is greater than $\psi$, which disobey the constraint $Pr_{\text{success}}^{\text{BF}} < \psi$. So, let

$$2\lambda_e \exp[-M/P_s]/P_s = \psi.$$
Under large SINR assumption, \((1 - M/P_s)2\lambda_0/P_s = \psi\), and we can get two solutions:

\[
\begin{align*}
\alpha &= \lambda_0 \left( 1 - \sqrt{1 - 2\psi M/\lambda_0} \right) / \psi, \\
\beta &= \lambda_0 \left( 1 + \sqrt{1 - 2\psi M/\lambda_0} \right) / \psi.
\end{align*}
\]

(C2.1.1) If \(M \leq 2\lambda_0\),
(C2.1.1.1) If \(\exp[-M/2\lambda_0] \leq \psi\),
then \(P_s = 2\lambda_0\);
(C2.1.1.2) If \(\exp[-M/2\lambda_0] > \psi\),
then \(P_s = \beta\);
(C2.1.2) If \(2\lambda_0 < M < P_{th}\),
(C2.1.2.1) If \(2\lambda_0 \leq \alpha < P_{th} < \beta\),
then \(P_s = \alpha\);
(C2.1.2.2) If \(2\lambda_0 \leq \alpha < \beta \leq P_{th}\),
then \(P_s = \alpha\) or \(\beta\);
(C2.1.2.3) If \(\alpha < 2\lambda_0 < \beta \leq P_{th}\),
then \(P_s = \beta\).
(C2.1.3) If \(M \geq P_{th}\),
(C2.1.3.1) If \(\exp[-M/P_{th}] \leq \psi\),
then \(P_s = P_{th}\);
(C2.1.3.2) If \(\exp[-M/P_{th}] > \psi\),
then \(P_s = \alpha\).

Other situations contradict one of the three constraints mentioned above: \(P_s \leq P_{th}\), \(P_s > 2\lambda_0\), and \(P_{s,\text{success}} > \psi\).

After \(P_s\) is derived, the maximal stable throughput of this system is \(\max\{\lambda_0\} = P_{s,\text{success}}^{\text{max}} / 2\).

After some transformation of the above proof process, Proposition 1 can be obtained.

References


Jin-Xin Niu was born in Heilongjiang, China. He is currently working towards the Ph.D. degree with the National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China (UESTC), Chengdu, China. His research interests include wireless communications and networks and cooperative communications systems.

Wei Guo was born in Sichuan, China. He received his B.S. and M.S. degrees from UESTC. Currently, he is a professor with the National Key Laboratory of Science and Technology on Communications, UESTC. His research interests include wireless communications and networks, satellite and space communications technology, software systems, and 5G technology.