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Abstract—Using a relaying system to provide spatial diversity and improve the system performance is a tendency in the wireless cooperative communications. Amplify-and-forward (AF) mode with a low complexity is easy to be implemented. Under the consideration of cooperative communication systems, the scenario includes one information source, $M$ relay stations and $N$ destinations. This work proposes a relay selection algorithm in the Raleigh fading channel. Based on the exhaustive search method, easily to realize, the optimal selection scheme can be found with a highly complicated calculation. In order to reduce the computational complexity, an approximate optimal solution with a greedy algorithm applied for the relay station selection is proposed. With different situations of the communication systems, the performance evaluation obtained by both the proposed algorithm and the exhaustive search algorithm are given for comparison. It shows the proposed algorithm could provide a solution approach to the optimal one.

Keywords—Amplify-and-forward mode, cooperative communication, exhaustive search, greedy algorithm, relay selection.

1. Introduction

The cooperative relaying scheme with a character of spatial diversity improves the system performance and becomes important in the modern wireless communications\(^1\)-\(^3\). Comparing with multiple carrier modulation schemes and multiple input multiple output (MIMO) scheme, it provides a high throughput performance. The destination user could receive data with a spatial diversity with employing the scheme. Even though the destination user has no multiple antennas, cooperative communication systems work with the character of spatial diversity. By employing the relay station as the function of antenna, it increases the transmission data rate and provides a reliable channel capacity\(^4\). Obviously, with a consideration of low cost, cooperative communication systems are a tendency in the future communications.

For the data transmission techniques, there are three strategies in cooperative communications\(^5\)-\(^6\). One is the amplify-and-forward (AF) mode, another is the decode-and-forward (DF) mode, and the other is the compress and forward (CF) mode. With the AF mode, the transmitted signal could be amplified and retransmitted to destination. It is easy to be implemented with a low complexity. For applications, it is good to those transmissions with a short distance. Similarly, with the DF mode, the relay station decodes and demodulates the received signal and, then, recodes and modulates the signal to retransmit to the destination. Within the CF mode, the relay station does not have to decode the compressed signal. It uses the coding schemes to compress the received signal and retransmit to the destination. In general, these three modes could effectively improve the system performance. Without the limitation of peer-to-peer transmission, the cooperative communication could obtain the advantage of system performance enhancement.

Among those three kind modes in the cooperative communications, the AF mode is with the characteristics of low complexity to be implemented\(^3\). Based on the AF mode, in this paper, a relay selection algorithm for downlink is proposed for the wireless cooperative systems. The algorithm is proposed based on the maximum amount of mutual information between the source and the destination. Relay selection could be an optimization by using an exhaustive search method. The exhaustive search method could be simply realized. However, the exhaustive search method is with a high computational complexity when the number of relay stations and the number of destination stations increase. Hence, an approximate optimal relay selection scheme with a greedy algorithm applied is proposed to reduce the computational complexity. The following section introduces the AF mode in the cooperative systems. It begins with the single user environment and, then, the multi-destination user environment. Under the different situations, the theoretical derivation for the mutual information between the source station and destination station is provided. The third section
describes the proposed relay selection algorithm. Also, the numerical results are shown for the comparison between the exhaustive search method and the proposed one. Finally, the conclusions are given in the last section.

2. Amplify-and-Forward Mode

In the cooperative communications, there are three elements in the system. One is the source station, another is relay station, and the other is the destination station. Each station has a transmitter, a receiver, and an antenna. It assumes that each station could not transmit and receive simultaneously. First, consider a single user environment, as shown in Fig. 1. $h_{s,d}$ is denoted as the channel response between the source station and destination station. Similarly, $h_{r,s}$ and $h_{s,d}$ are denoted as channel response between the source station and the relay station and channel response between the relay station and destination station, respectively.

![Fig. 1. AF mode transmission model.](image)

At the first time instant, the received signal at relay station and destination station from the source station could be expressed as

$$y_{s,r} = \sqrt{P_s} h_{s,r} x + n_{s,r}$$

$$y_{s,d} = \sqrt{P_s} h_{s,d} x + n_{s,d}$$

where $P_s$ is the signal power from the source station, $x$ is the transmitted signal from the source station, and $n_{s,r}$ and $n_{s,d}$ are additive white Gaussian noise (AWGN) with the variance $N_0$. In the AF mode, the relay station amplifies the received signal and retransmits the amplified signal to the destination station. At the destination station, the antenna receives the signal with two diversity components. For normalization, the signal gain from the relay station is defined as $\beta$:

$$\beta_r = \frac{\sqrt{P_s}}{\sqrt{P_s h_{s,r}^2 + N_0}}$$

where $P_s$ is the signal power from the relay station. In order to evaluate the signal to noise ratio (SNR) at the instant time, $\text{SNR}_{s,r} = P_s |h_{s,r}|^2 / N_0$ represents the SNR between the source station and the relay station, $\text{SNR}_{s,d} = P_s |h_{s,d}|^2 / N_0$ between the source station and relay station, and $\text{SNR}_{r,s} = P_s |h_{r,s}|^2 / N_0$ between the relay station and the destination station.

At the secondary time instant, the received signal at destination station from the relay station could be expressed as

$$y_{r,d} = \frac{\sqrt{P_r}}{\sqrt{P_r h_{r,d}^2 + N_0}} h_{s,r} y_{s,r} + n_{r,d}$$

where $n_{r,d}$ is AWGN with the variance $N_0$. Insert (1) to (4), the equation could be derived as

$$y_{r,d} = \frac{\sqrt{P_r}}{\sqrt{P_r h_{r,d}^2 + N_0}} \sqrt{P_s h_{s,r} h_{s,d}} + n'_{r,d}$$

where $n'_{r,d}$ is AWGN with the variance $N_0'$.

At the destination station, the received signal has two components. One is from the source station and the other is from the relay station. With the processing of maximal ratio combiner, $a_1$ and $a_2$ are the separated gains of source-to-destination link and relay-to-destination link, the receiver could obtain the optimal gains from those two stations, and the received signal $y$ could be expressed as

$$y = a_1 y_{s,d} + a_2 y_{r,d}$$

where

$$a_1 = \frac{\sqrt{P_r h_{s,r}^2}}{N_0}$$

$$a_2 = \frac{\frac{\sqrt{P_s}}{\sqrt{P_r h_{r,d}^2 + N_0}} \sqrt{P_s h_{s,d}}}{\frac{P_r h_{r,d}^2}{P_s h_{s,d}^2} + 1 + \frac{1}{N_0}}$$

Hence, the maximum SNR at the instant time could be approached as

$$\text{SNR}_{\text{max}} = \frac{P_s |h_{s,d}|^2}{N_0} + \frac{1}{N_0} \frac{P_s |h_{s,d}|^2}{P_r |h_{r,d}|^2 + P_r |h_{r,d}|^2 + N_0}$$

Then, according the Shannon theory, the channel capacity $C$ could be expressed as

$$C = \text{BW} \log_2 (1 + \text{SNR}_{\text{max}})$$

where BW is the channel bandwidth and $I_{AF}$ is the mutual information, in AF mode, between the source station and the destination station. Hence, in the normalized channel, the maximum instantaneous mutual information could be found by maximizing the equation

$$C = \frac{1}{2} \log_2 (1 + \text{SNR}_{\text{max}})$$

That is, the maximize mutual information between the source and the destination becomes

$$\max(I_{AF}) = \frac{1}{2} \log_2 \left[ 1 + \frac{P_s |h_{s,d}|^2}{N_0} + \frac{1}{N_0} \frac{P_s |h_{s,d}|^2}{P_r |h_{r,d}|^2 + P_r |h_{r,d}|^2 + N_0} \right]$$

(13)
Then, consider a multiple user environment where there are one source station, $M$ relay stations, and $N$ destination stations\(^7\), as shown in Fig. 2. $R = \{r_1, r_2, \ldots, r_M\}$ and $D = \{d_1, d_2, \ldots, d_N\}$ are defined as the set of relay stations and the set of destination stations, respectively. $N$ is less than $M$. Under rayleigh fading environment, $h_{r,i}$ and $h_{d,i}$ are denoted as the channel response between the source station and the relay station $i$ and the channel response between the source station and the destination station $i$, respectively. $h_{r,j}$ is defined as the channel response between the relay station $i$ and the destination station $j$. With the similar situation in the single user environment, the source station transmits the signal to $N$ destination stations. At the relay station $i$ and the destination station $j$, the received signals from the source station are

\[
y_{s,i} = \sqrt{P_s} h_{s,i} x_j + n_i \quad (14)
\]

\[
y_{s,dj} = \sqrt{P_s} h_{d,i} x_j + n_d \quad (15)
\]

At the second time instant, the received signal at the destination station $j$ from the relay station $i$ is

\[
y_{r,dj} = \sqrt{P_s} \frac{h_{r,i}}{\sqrt{P_s} h_{r,i} + N_0} h_{s,i} y_i + n_i' \quad (16)
\]

where $n_i'$ is AWGN with the variance $N_0'$:

\[
N_0' = \left( \frac{P_s}{P_s h_{r,i}^2 + N_0} + 1 \right) N_0 \quad (17)
\]

Then, with an optimal choosing the relay station $i$, similarly to (13), the instantaneous mutual information between the source station and the destination station $j$ could be obtained by maximizing the equation

\[
I_{M,i,j} = \frac{1}{2} \log_2 \left( 1 + \frac{P_s h_{r,i}^2}{N_0} + \frac{P_s P_h h_{d,i}^2}{N_0^2} + \frac{P_s P_h h_{r,i} h_{d,i}}{N_0^2} + \frac{P_s}{N_0} h_{r,i}^2 + \frac{P_s}{N_0} h_{d,i}^2 + N_0 \right) \quad (18)
\]

Hereby, how to choose the appropriate relay station $i$ to approach the maximum mutual information becomes an important issue. The relay selection algorithm is proposed in the following section.

### 3. Relay Selection Algorithm

Under the above situation, considering each destination station receiving the signal from one relay station only, this work intends to find the pairs to match these $N$ destination stations and the corresponding relay station. Hence, the limitation to this problem could become

\[
\max_{\rho_{r,i}, \rho_{d,j}} \sum_{i=1}^{M} \sum_{j=1}^{N} \rho_{r,i} I_{i,j} \quad (19)
\]

under the conditions:

\[
\sum_{i=1}^{M} \rho_{r,i} = 1, \quad \forall i = 1, 2, \ldots, N \quad (20)
\]

\[
\sum_{j=1}^{N} \rho_{d,j} = 1, \quad \forall i = 1, 2, \ldots, M \quad (21)
\]

where $\rho_{r,i}$ is defined as the connection between relay station $i$ and the destination station $j$. In (20), for each destination belonging to $D$, there is only one corresponding relay station connected to the destination station. $\rho_{r,i} = 1$ when there is a connection between the relay station $i$ and the destination station $j$ and $\rho_{r,i} = 0$ for other situations. In (21), for each relay station belonging to $R$, the relay station could connect at most one destination station only. To achieve the optimal solution to (19), the exhaustive search method could be employed.

With the exhaustive search method, the algorithm should begin to calculate (19).

First, for each relay station, calculate $|h_{r,i}|^2$ and $|h_{d,i}|^2$.

Second, calculate $I_{d,j}$ in (19) and recode it in an $M \times N$ array such as

\[
\begin{array}{cccccc}
i & j & d_1 & d_2 & \ldots & d_N \\
1 & I_{1,1} & I_{1,2} & \ldots & I_{1,N} \\
2 & I_{2,1} & I_{2,2} & \ldots & I_{2,N} \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
M & I_{M,1} & I_{M,2} & \ldots & I_{M,N} \\
\end{array}
\]

Third, iteratively calculate the $(i, j)$ combination for the maximum value in (19) under the limitation of (20) and (21). The optimal mapping for the relay station and destination station could be found with calculation within $\binom{M}{N} N!$ combinations.

It is with a high complicated computing to implement. Specially, the complexity of computation is high for the
burst traffic arriving. In order to reduce the computational complexity, a greedy algorithm applied for relay selection scheme (called sub-optimal algorithm) with a maximum mutual information finding is proposed. The advantage to use a greedy algorithm is that the solutions to the optimal problem can be straightforward and easy to understand\(^9\). Considering the cooperative system mentioned above, the relay selection algorithm could be modified from the exhaustive search method.

Step 1: similarly, calculate \( |h_{i,j}|^2 |h_{j,i}|^2 \) for all relay stations.

Step 2: create an \( M\times N \) array, calculate \( I_{i,j} \) and recode it in the array as same as the second step in the exhaustive search method.

Step 3: according to the created array, assign the maximum value of \( I_{i,j} \) to the pair of relay station \( i \) and destination station \( j \). Randomly choose one of maximum values to assign if there are more than one maximum values appeared. The pair \((i,j)\) represents that the \( i \)th relay station is assigned to serve the \( j \)th destination station. At the mean time, corresponding to the chosen cell, delete all other row cells in the array. It means the chosen relay station could not serve other destination stations. Similarly, corresponding to the chosen cell, delete all other column cells in the array. It means the chosen destination is served by one relay station only. Then, a new reduced matrix is created.

Step 4: repeat Step 3 till finishing the assignment work.

Step 5: according to the pair assignment, calculate the mutual information \( I_{AF} \).

With different relay station selection schemes, the performance based on the throughput of system is evaluated for different numbers of destinations. Fig. 1 and Fig. 2 show the channel capacity comparisons for different selection schemes and different number of the relay stations and destinations, respectively.

In Fig. 2, when the number of relay stations and destination stations are equal, there is no big difference with the calculation by using these two algorithms. Although the increasing capacity could be found when the number of relay station increases, the computational complexity based on the number of calculation is increasing a lot. The proposed greedy applied algorithm could effectively reduce the complicated calculation and trade off with less channel capacity.

In Fig. 3, when the number of relay station is larger than that of destination stations, there is less difference between the performances of these two algorithms. When the number of relay stations increases, the performance evaluation with the proposed greedy applied algorithm is closer to the optimal solution. It means the optimal relay station selection could be easy where more available relay stations are existed. However, when the number of relay station is increasing, the computational complexity is increasing. With the proposed greedy applied algorithm, it reduces a lot of complex calculations.

4. Conclusions

With a character of low cost, the cooperative communication system is a tendency in the future communications. In this application, it employs the relay station to increase the SNR at the destination station. However, in the multi-user environment, the pair matching of the corresponding relay station and destination station becomes complex if the number of relay stations and destination stations increase. Although the exhaustive search method could achieve the optimal solution to the problem, its complexity of computation could be a difficulty. This paper proposes the relay selective algorithm with a greedy algorithm applied for reducing the computational complexity. In this paper, the assumption of relay station assignment is based on the pair with one relay station and one destination station only. However, this assumption is to simplify the assignment scheme. In practical applications, more complicate relay station assignments should be detailed considered in the future. For example, the power control and the bandwidth allocation issues on the relay station could be the extension research on the cooperative communications.
References


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