Abstract—By utilizing total magnetic flux \( \phi \) of the primary and secondary windings of the flyback transformer as a state variable, the discrete-time model of current-mode controlled flyback converter is established, upon which the bifurcation behaviors of the converter are analyzed and two boundary classification equations of the orbit state shifting are obtained. The operation-state regions of the current-mode controlled flyback converter are well classified by two boundary classification equations. The theoretical analysis results are verified by power electronics simulator (PSIM). The estimation of operation-state regions for the flyback converter is useful for the design of circuit parameters, stability control of chaos, and chaos-based applications.

Index Terms—Bifurcation, chaos, discrete-time model, flyback converter.

1. Introduction

Bifurcation phenomena and border collision have been studied in buck, boost, and buck-boost converters\[1\]–[3]. In recent years, these phenomena have been also studied in peak-current-controlled superbuck converter\[4\], single-inductor dual-switching dc-dc converter\[5\], and boost PFC (power factor correction) converter\[6\]. However, few studies have been performed on isolated converter, such as flyback converter\[7\],[8]. In flyback converter, the secondary winding current of the transformer is zero when the switch is turned on, whereas the primary winding current of the transformer is zero when the switch is turned off. Thus the inductance currents of the primary and secondary windings of the transformer are discontinuous and can not be regarded as state variables to analyze the bifurcation behaviors. For this reason, total ampere-turns of the primary and secondary windings of the transformer have been presented as one of the variables to analyze the bifurcation behaviors on voltage-mode controlled flyback converter\[8\]. In this paper, we consider the total magnetic flux of the primary and secondary windings as a state variable to describe the dynamics of current-mode controlled flyback converter.

In current-mode controlled switching dc-dc converters, the operation-states may shift from continuous conduction mode (CCM) to discontinuous conduction mode (DCM), and three operation-state regions classified by stable period region, robust chaos region in CCM, and intermittent chaos region in DCM exist in the current-mode controlled switching dc-dc converters with the variations of circuit parameters, such as input voltage, output voltage, or reference current\[4\],[9]–[11]. Some methods have been developed to locate the boundary of the period-one zone of switching dc-dc converters\[12\]–[14], and an approach to locate two boundaries of three operation-state regions of current-mode controlled switching dc-dc converters has been proposed recently\[11\]. In this paper, the dynamics of a current-mode controlled flyback converter is analyzed through a discrete-time map model covering both CCM and DCM, and two boundary classification equations dividing three smooth operation-state regions over the parameter space of the converter are derived.

2. Modeling of Current-Mode Controlled Flyback Converter

2.1 Flyback Converter with Current-Mode Control

The schematic of current-mode controlled flyback converter is shown in Fig. 1, in which the main circuit topology is a second-order circuit consisting of an inductor L, a capacitor C, a switch S, a diode D, a load resistor R and a transformer.

![Fig. 1. Current-mode controlled flyback converter.](image-url)
The parameters of the transformer in Fig. 1 are the primary winding leakage inductance $L_1$, the secondary winding leakage inductance $L_2$, and the turn ratio of the primary and secondary windings $N_1$: $N_2$. A timer generates a free-running clock which controls the operation of current-mode control loop. The converter is controlled by a feedback loop consisting of a comparator and a RS trigger.

Fig. 2 shows the waveforms of the currents and the corresponding total magnetic flux of the primary and secondary windings of the flyback transformer. The switch $S$ is turned on at the beginning of each switching cycle, and during the time interval when the switch $S$ is turned on, the primary winding current $i_1$ rises linearly and the secondary winding current $i_2$ equals to zero. The switch $S$ is turned off when $i_1$ increases to a reference current $I_{ref}$, and during the time interval when the switch $S$ is turned off, $i_1$ equals to zero and $i_2$ decreases. In CCM, $i_2$ is always non-zero, while in DCM, $i_2$ decreases to zero during the time interval when the switch $S$ is turned off and remains at zero until the end of switching cycle. Because the switching frequency of flyback converter is usually much higher than the natural frequency of the converter, the dynamics of the inductor current waveform becomes piecewise linear.

### 2.2 State Equations

Since two currents $i_1$ and $i_2$ are discontinuous and can not be regarded as state variables\(^8\), we consider the total magnetic flux $\phi$ of the primary and secondary windings as a state variable to describe the dynamics of the flyback converter. The expression of $\phi$ is

$$\phi(t) = \psi_1/N_1 + \psi_2/N_2 = L_1i_1/N_1 + L_2i_2/N_2 \quad (1)$$

where $\psi_1$ and $\psi_2$ are the magnetic flux linkages of the primary and secondary windings of the transformer, respectively.

The converter can be regarded as a system with a variable structure that toggles its topologies according to the states of the switches as shown in Fig. 2. Typically, when the converter operates in DCM, three switch states can be identified as follow:

- **State 1**: switch $S$ on and diode $D$ off.
- **State 2**: switch $S$ off and diode $D$ on.
- **State 3**: switch $S$ off and diode $D$ off.

When the converter operates in CCM, only two switch states are identified by State 1 and State 2, i.e., the state 3 does not appear in CCM. Thus, the system operating in both CCM and DCM can be described by the following state equations.

**State 1**: When $S$ is on, $D$ is off, $i_2 = 0$, $\phi = L_1i_1/N_1$ and $d\phi/dt = (L_1/N_1)di_1/dt$. The state equation is

$$L_1di_1/dt = E, \quad d\phi/dt = E/N_1. \quad (2)$$

**State 2**: When $S$ is off, $D$ is on, $i_1 = 0$, so $\phi = L_2i_2/N_2$ and $d\phi/dt = (L_2/N_2)di_2/dt$. The relationship between the primary inductor $L_1$ and the secondary inductor $L_2$ is $L_1/L_2 = N_1^2/N_2^2$. Thus the state equation is

$$L_2di_2/dt = -V_o, \quad d\phi/dt = -V_o/N_2. \quad (3)$$

**State 3**: When the converter operates in DCM, both $S$ and $D$ are off, $i_1 = i_2 = 0$. The state equation is

$$di_1/dt = 0, \quad d\phi/dt = 0. \quad (4)$$

### 2.3 Two Borders

For current-mode controlled flyback converter operating in DCM, there are two borders in the discrete state-space. The border $\phi_{b1}$ is defined as the total magnetic flux of the primary and secondary windings at the beginning of switching cycle when the current $i_1$ reaches $I_{ref}$ just at the end of the switching cycle. The border $\phi_{b2}$ is defined as the total magnetic flux of the primary and secondary windings at the beginning of switching cycle which decreases to zero just at the end of the switching cycle. Fig. 3 shows the total magnetic flux waveforms of current-mode controlled flyback converter at these borders. Fig. 3 (a) shows the evolution of total magnetic flux if $\phi_{b1} = \phi_{b1}$, where the current $i_1$ reaches $I_{ref}$ at the end of the switching cycle, and the switch $S$ remains on throughout this switching cycle. Fig. 3 (b) shows the evolution of total magnetic flux if $\phi_{b2} = \phi_{b2}$, where the current $i_2$ decreases to zero at the end of the switching cycle.

Based on the definitions of these two borders, we can easily obtain two borders of the total magnetic flux, $\phi_{b1}$ and $\phi_{b2}$, as below:

$$\phi_{b1} = (L_1I_{ref} - ET)/N_1 \quad (5)$$

Fig. 3. Total magnetic flux waveforms of current-mode controlled flyback converter with two borders: (a) $\phi_a = \phi_{b1}$, $\phi_{b1+1} = L_1I_{ref}/N_1$ and (b) $\phi_a = \phi_{b2}$, $\phi_{b1+1} = 0$. 

![Fig. 2. Waveforms of the inductance currents and the magnetic flux.](image-url)
\[
\phi_{b2} = \left( \frac{N_2 E}{N_1 V_o} + 1 \right) \frac{L_i E}{N_1} - \frac{ET}{N_1}.
\] (6)

2.4 Discrete-Time Model

With these two borders given by (5) and (6), there are three types of orbits between consecutive clock instants.

1) \( \phi_b \leq \phi_{b1} \). The switch remains on throughout the switching cycle, and the map is easily derived from (2) and given by

\[
\phi_{n+1} = \phi_n + ET / N_1. \tag{7}
\]

2) \( \phi_{b1} < \phi_n < \phi_{b2} \). The magnetic flux of the primary winding increases to \( L_i I_{ref}/N_1 \) and then the magnetic flux of the secondary winding decreases until the end of the switching cycle, and the map is derived from (3) and given by

\[
\phi_{n+1} = -\frac{N_1 V_o}{N_2 E} (\phi_n - \phi_{b2}). \tag{8}
\]

3) \( \phi_b \geq \phi_{b2} \). The inductance current \( i_2 \) of the secondary winding decreases to zero during the \( n \)th switching cycle, i.e., the converter enters DCM. Thus, at the end of the \( n \)th switching cycle, we have

\[
\phi_{n+1} = 0. \tag{9}
\]

Thus, the discrete-time model of current-mode controlled flyback converter can then be written in the form:

\[
\begin{cases}
\phi_n + ET / N_1, & \phi_b \leq \phi_{b1}; \\
-\frac{N_1 V_o}{N_2 E} (\phi_n - \phi_{b2}), & \phi_{b1} < \phi_n < \phi_{b2}; \\
0, & \phi_b \geq \phi_{b2}.
\end{cases} \tag{10}
\]

Based on three piecewise linear equations containing two borders, the discrete map model of current-mode controlled flyback converter is obtained and the corresponding dynamical analysis can be performed.

3. Dynamics and Operation-State Estimation

3.1 Dynamics of Current-Mode Controlled Flyback Converter

By utilizing the discrete-time model (10), the bifurcation to chaotic behavior of current-mode controlled flyback converter with circuit parameters variation can be effectively exhibited. The numerical simulations are performed by using MATLAB software platform with following parameters: \( I_{ref} = 1.2 \text{ A}, L_1 = 1 \text{ mH}, T = 100 \mu s \), and \( N_1:N_2 = 3:2 \). If we fix \( E = 7 \text{ V} \) and take \( V_o \) as bifurcation parameter, we can obtain the magnetic flux bifurcation diagram with the increasing of \( V_o \) as shown in Fig. 4 (a). If we fix \( V_o = 9 \text{ V} \) and take \( E \) as bifurcation parameter, then we can obtain the magnetic flux bifurcation diagram with the increasing of \( E \) as shown in Fig. 4 (b). In Fig. 4, two borders \( \phi_{b1} \) and \( \phi_{b2} \) are plotted by using the dashed line and dash-dot line, respectively.

From Fig. 4, it is found that the current-mode controlled flyback converter has complex dynamical behaviors. First considering \( V_o \) as variable, when \( V_o \) increases gradually, a period-doubling bifurcation occurs at \( V_o = 4.67 \text{ V} \). After the occurrence of period-doubling bifurcation, the unstable periodic orbit with period-two collides with the border \( \phi_{b1} \), resulting in a border collision bifurcation at the same parameter value, and the operation-state of the converter goes into the robust chaos with CCM from the stable period. When \( V_o \) increases further, the chaotic orbit collides with the border \( \phi_{b3} \) at \( V_o = 8 \text{ V} \), resulting in another border collision bifurcation, and the operation-state of the converter shifts into the intermittent chaos with DCM from the robust chaos with CCM. Then considering \( E \) as variable, when \( E \) decreases gradually, a reverse period-doubling bifurcation occurs at \( E = 13.5 \text{ V} \). After the occurrence of period-doubling bifurcation, the converter operates in DCM, i.e., the current \( i_2 \) remains at zero in the durations of some switching cycles, the unstable periodic orbit collides with the border \( \phi_{b2} \), resulting in a border collision bifurcation at the same parameter, and the operation-state of the converter directly enters into sub-harmonics with DCM. When \( E \) decreases further, the unstable periodic orbit collides with the border \( \phi_{b1} \) at \( E = L_i I_{ref}/T = 12 \text{ V} \), resulting in the non-zero periodic orbit to be folded. The folded non-zero periodic orbit collides with the border \( \phi_{b2} \) at \( E = 10.8 \text{ V} \), and a period-four orbit emerges from the period-two orbit. The operation-state of the converter jumps into the intermittent chaos with DCM from the period-sixteen orbit at \( E = 8.17 \text{ V} \).
The above analysis results show that there exist three operation-state regions, i.e., stable period region, robust chaos region with CCM, and intermittent chaos region with DCM, in current-mode controlled flyback converter. Especially, the super-stable periodic orbits exist in DCM due to the occurrence of zero eigenvalue\(^9\). The converter shows weak chaos and strong intermittency, which means that the chaotic behavior becomes weak in DCM\(^10\).

### 3.2 Boundary Classification Equations for Operation-State

From (10), the eigenvalue \(\lambda\) of the characteristic equation for current-mode controlled flyback converter is given by \(\lambda = -N_1 V_o/N_2 E\). To ensure stable operation, \(\lambda\) must fall between \(-1\) and 1\(^{15}\). The first period-doubling occurs when \(\lambda = -1\). Hence, by putting \(\lambda = -1\), the first boundary classification equation \(\sigma_1\) for the operation mode shifting from the stable period-one to subharmonics and chaos will be

\[
\sigma_1 = N_2 E - N_1 V_o = 0
\]

which implies that if the circuit parameters satisfy \(\sigma_1 > 0\), i.e., \(V_o < N_2 E/N_1\), the converter operates with periodic oscillation, otherwise with subharmonics or chaotic oscillation.

It is clear that the maximum total magnetic flux at the end of the \(n\)th switching cycle is \(\varphi_{n+1, \text{max}} = L_1 I_{\text{ref}}/N_1\). When the total magnetic flux of the flyback converter reaches the border \(\varphi_{b2}\), the border collision bifurcation and operation-state shift occur. Under this condition, there exists \(\varphi_{b2} = \varphi_{n+1, \text{max}}\), i.e.,

\[
\left(\frac{N_1 E}{N_1 V_o} + 1\right) \frac{L_1 I_{\text{ref}}}{N_1} - \frac{ET}{N_1} = \frac{L_1 I_{\text{ref}}}{N_1}.
\]

Therefore, the second boundary classification equation \(\sigma_2\) for the operation-state region shifting from CCM to DCM will be

\[
\sigma_2 = N_2 L_2 I_{\text{ref}} - N_1 V_o T = 0.
\]

If the circuit parameters satisfy \(\sigma_2 < 0\), i.e., \(V_o > N_2 L_2 I_{\text{ref}}/N_1 T\), the converter will operate in DCM, otherwise in CCM.

It is remarkable that \(\sigma_1\) only depends on the input voltage \(E\), the output voltage \(V_o\), and the turns ratio of the primary and secondary windings of the transformer, while \(\sigma_2\) depends on all of circuit parameters of current-mode controlled flyback converter except for the input voltage \(E\).

### 3.3 Estimation of Operation-State Regions

Considering the circuit parameters with ranges \(E = 2\sim18\) V and \(V_o = 2\sim12\) V, and letting \(I_{\text{ref}} = 1.2\) A, \(L_1 = 1\) mH, \(T = 100\) \(\mu\)s, and \(N_1:N_2 = 3:2\), we can obtain the parameter space map as shown in Fig. 5 (a). The higher periodicities are depicted with deeper gray levels, the darker shade areas imply chaos, while the white and shallower gray areas mean low period.

The three smooth operation-state regions of current-mode controlled flyback converter over the parameter space can be divided by above boundary classification equations \(\sigma_1\) and \(\sigma_2\). Fig. 5 (b) shows the regions of the orbit operation-states corresponding to Fig. 5 (a). From (11) and (12), the two boundaries are \(\sigma_1: V_o = N_2 E/N_1\) and \(\sigma_2: V_o = N_2 L_1 I_{\text{ref}}/N_1 T\) respectively. The boundary \(\sigma_1\) is called as the first period-doubling bifurcation borderline and the boundary \(\sigma_2\) is called as the operation mode shifting borderline. The regions of stable period-one, robust chaos in CCM, and intermittent chaos in DCM are shown in Fig. 5 (b). From the parameter space map, the different operation-state regions of the converter can be demonstrated clearly.

It is visible that the 1-D bifurcation diagrams of \(\varphi_n\) versus \(V_o\) and \(E\) in Fig. 4 (a) and Fig. 4 (b) can be obtained along the paths from point A to point B and from point C to point D in Fig. 5 (a), respectively. It should also be noted that the boundary classification equation \(\sigma_2\) will lose its physical significance when current-mode controlled flyback converter is located in stable operation-state region.

### 3.4 PSIM Simulation Results

In order to verify theoretical analysis, the PSIM (power electronics simulator) simulations of the current-mode controlled flyback converter are performed with the parameters as mentioned above, by first fixing \(E = 7\) V and then \(V_o = 9\) V. With the increasing of \(V_o\) or \(E\), the waveforms of the primary winding current \(i_1\) and the secondary winding current \(i_2\) of the flyback transformer are obtained in Fig. 6 and Fig. 7. Fig. 6 depicts that with the variation of output voltage \(V_o\), 4 V, 4.68 V, 7 V and 10 V, the period-1 with CCM, period-2 with CCM, robust chaos with CCM, and intermittent chaos with DCM occur respectively. Fig. 7 describes that with the variation of input...
voltage $E$: 5 V, 9.5 V, 12 V, and 14 V, the intermittent chaos with DCM, period-4 with DCM, period-2 with DCM, and period-1 with CCM occur respectively. These simulation results are consistent with those results shown in Fig. 4.

Considering the other four sets of circuit parameters located in the different operation-state regions divided by two boundary classification equations $\sigma_1$ and $\sigma_2$ in Fig. 5 (b), we can further obtain the simulation results as shown in Fig. 8. Fig. 8 (a) shows that when the circuit parameters are located in CCM robust chaos region in Fig. 5 (b), the converter is in chaotic state with CCM. Fig. 8 (b) depicts that when the circuit parameters are just located at the boundary $\sigma_2$ in Fig. 5 (b), the secondary winding current $i_2$ decreases to zero at the end of some switching cycle, and the operation-state shifts between CCM and DCM. The circuit parameters for Fig. 8 (c) are located in DCM intermittent chaos region in Fig. 5 (b), thus the converter is in chaotic state with intermittency. While the circuit parameters locating in DCM intermittent chaos region in Fig. 5 (b) are selected, the converter operates at period-2 with DCM, as shown in Fig. 8 (d).

![Fig. 6. Simulation waveforms while $V_o$ is increased along the path from point A to point B in Fig. 5 (a): (a) CCM, period-1 for $V_o = 4$ V, (b) CCM, period-2 for $V_o = 4.68$ V, (c) CCM, robust chaos for $V_o = 7$ V, and (d) DCM, intermittent chaos for $V_o = 10$ V.](image)

![Fig. 7. Simulation waveforms while $E$ is increased along the path from point C to point D in Fig. 5 (a): (a) DCM, intermittent chaos for $E = 5$ V, (b) DCM, period-4 for $E = 9.5$ V, (c) DCM, period-2 for $E = 12$ V, and (d) CCM, period-1 for $E = 14$ V.](image)

![Fig. 8. Simulation waveforms while $E$ and $V_o$ are arbitrarily selected in different operation-state regions of Fig. 5(b): (a) CCM, robust chaos for $E = 3$ V and $V_o = 6$ V, (b) shifting between CCM and DCM, critical robust chaos for $E = 6$ V and $V_o = 8$ V, (c) DCM, intermittent chaos for $E = 5$ V and $V_o = 10$ V, and (d) DCM, period-2 for $E = 12$ V and $V_o = 11$ V.](image)

### 4. Conclusions

With the variations of circuit parameters such as input voltage and output voltage, the current-mode controlled flyback converter exhibits two borders and its operation-states can shift between stable period-1, robust chaos in CCM, and intermittent chaos in DCM via period-doubling bifurcation and border collision bifurcation. Two boundary classification equations of operation-state regions can characterize the constitutive relations that the current-mode controlled flyback converter shifts among different operation-states. Utilizing total magnetic flux $\phi$ of the primary and secondary windings as a state variable, we establish the discrete-time model with two borders, analyze the bifurcation behaviors, and obtain two boundary classification equations. The estimated operation-states by two boundary classification equations are verified by PSIM simulation results of the current-mode controlled flyback converter.

### References


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