Carrier Synchronization for Very Low SNR

Jun-Hui Xu, Zhao Wang, Zhong-Jie Gao, and Zhong-Pei Zhang

Abstract—The Turbo decoding performance will suffer serious degradation under low signal-to-noise ratios (SNR) conditions for the reason of residual frequency and phase offset in the carrier. In this paper, an improved residual carrier frequency offset estimation algorithm based on a priori probability aided (APPA) phase estimation is proposed. A carrier synchronization loop that combines the iterative Turbo decoder and the phase estimator together is constructed, where the extrinsic information obtained from the Turbo decoder is used to aid an iterative phase estimation process. The simulation results show that the algorithm performs successfully under very low SNR conditions (for example, less than $-7.4$ dB) with large frequency offset and phase error and the performance of this algorithm is very close to the optimally synchronized system.

Index Terms—Carrier synchronization, frequency offset, phase error, signal-to-noise ratios (SNR).


1. Introduction

Turbo codes and low density parity check (LDPC) can approach the Shannon channel capacity of the system and operate at very low symbol-to-noise ratios (SNR)$^{[1][2]}$. But at very low SNR, the residual frequency and phase offset will make obvious degradation on the Turbo decoding performance. For this reason carrier synchronization schemes are necessary. However, in this situation, accurate carrier recovery is a challenge. For example, with a rate 1/6 code and binary phase-shift keying (BPSK) modulation, few radio receivers can find and track the symbol timing and frequency offset when $E_b/N_0<-7.8$ dB$^{[3]}$. In this paper, we propose an improved a priori probability aided (APPA) phase estimation algorithm to estimate the carrier frequency offset under very low SNR conditions.

The traditional phase estimation algorithms including the non-code-aided (NCA) algorithms ignore the information from the error-control coding and assume that the transmitted symbols are mutually independent. In recent years, a code-aided carrier synchronization algorithm which can work at very low SNR was widely researched. In [4] to [6], an iterative phase estimator scheme based on expectation-maximization (EM) algorithm was proposed. Another code-aided scheme using the soft information of the decoder to aid phase error estimate was proposed in [7]. In [8], the authors introduced an iterative soft-decision -directed strategy based on training sequence. A costas loop algorithm (CL) was proposed in [9], which used the extrinsic information obtained from the turbo decoder to aid an iterative phase estimation process, and then fed the error signal to the loop filter and numerically controlled oscillators (NCO) to generate the signal for de-rotated. The simulation results showed that the CL algorithm can work at very low SNR. In [10], an algorithm based on APPA was proposed, and it was a maximum-likelihood (ML) strategy which could work at very low SNR yet with no frequency offset$^{[9]}$. In this paper, we propose an improved APPA algorithm which is suitable for the condition that there exists a small frequency offset. And the simulation results show that the algorithm performs well when existing residual frequency and phase offsets at very low SNR and its performance is very close to the optimally synchronized system.

Considering the limitation of the APPA algorithm when the frequency offset is large, we set up a frequency-sweep module to track the large frequency offset. According to the simulation results, the maximum phase offset that can be rectified by this improved APPA algorithm is 80 degrees.

This paper is organized as follows. Section 2 illustrates the system model for code-aided carrier synchronization. Section 3 and Section 4 describe the improved APPA algorithm and the frequency-sweep module in detail, respectively. Section 5 gives the simulation results. Finally, conclusions are given in Section 6.

2. System Model

Fig. 1 shows the carrier synchronization loop in the receiver. There are two loops in the system illustrated in Fig. 1. The outer loop in Fig. 1 is frequency-search loop, while the inner loop is the phase-track loop. This system works in 2 steps. At first, we start the frequency-search loop to track the large frequency offset. When the frequency offset is locked by the search module, the system switches on the phase-track loop to rectify the phase offset.

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In this paper a rate 1/6 Turbo code is used\cite{3}, and we perform 36 iterations in the Turbo decoder. It uses binary-shift keying modulation (BPSK) in the transmitter. After coding, the length of the code word is 2400 bits.

After frequency offset search, the received signal $r_i$ can be represented as

$$r_i = C_i \exp(j2\pi k_i \Delta f T_s + \theta_i) + n_i = A_i \exp(j\phi)$$  \hspace{1cm} (1)

where $C_i$ is the power of the BPSK symbol, $A_i$ is the magnitude of $r_i$, $\Delta f$ represents the residual frequency, $T_s$ is the period of a symbol, and $\theta_i$ is the phase that needs to estimate. We assume an additive white Gaussian noise (AWGN) channel with a noise power of $\sigma^2$. \{ $n_i$ \} are the samples of this AWGN channel.

Let $\hat{\theta}$ denote the estimation value of $\theta_i$, $y_i$ represent the signal after the phase rectify. Then $y_i$ can be written as

$$y_i = r_i \exp(-j\hat{\theta}) .$$  \hspace{1cm} (2)

3. Code-Aided Carrier Synchronization Based on Improved APPA Algorithm

Assume $\phi = \phi_0 + 2\pi i \Delta f$ to represent the instantaneous carrier phase, which includes two parameters: constant phase offset $\phi_0$ and frequency offset $\Delta f$. The log-likelihood function of $i$th symbol can be represented by

$$A'_i(\phi) = \log \left\{ \frac{1}{2\pi} \sum_{m=0}^{1} P_i(m) \exp \left[ \frac{1}{\sigma^2} \text{Re} \left( r_i s_i^*(\phi) \right) \right] \right\} ,$$ \hspace{1cm} (3)

where $P_i(m)$ represents the probability of each possible transmitted symbol, indicating that the $i$th transmitted signal is the $m$th constellation point.

In BPSK, there are two possible transmitted symbol values

$$\begin{align*}
x(0, \phi) &= -\exp(j\phi) \\
x(1, \phi) &= \exp(j\phi).
\end{align*}$$  \hspace{1cm} (4)

In binary transmission, we can get this equation $P(d_i = 1) + P(d_i = 0) = 1$, and hence, we can compute the priori probabilities from $L(i)$ by

$$\begin{align*}
P(d_i = 1) &= \frac{\exp(L(i))}{1 + \exp(L(i))} \\
P(d_i = 0) &= \frac{1}{1 + \exp(L(i))}
\end{align*}$$  \hspace{1cm} (5)

where $L(i)$ is the extrinsic information which can be obtained from the Turbo decoder. Substituting (4) and (5) to (3), we can get the LLF of $i$th symbol for BPSK system

$$A'_{BPSK}(\phi) = \log \left\{ \text{sech} \left( \frac{L(i)}{2} \right) \cosh \left[ \frac{L(i)}{2} + \frac{A_i \cos(\phi - \phi_i)}{\sigma^2} \right] \right\} .$$  \hspace{1cm} (6)

We expand (4) as a Fourier series

$$A'_{BPSK}(\phi) = a_0' + \sum_{n=1}^{\infty} \left[ a_n' \cos(n(\phi - \phi_i)) + b_n' \sin(n(\phi - \phi_i)) \right] .$$  \hspace{1cm} (7)

The Fourier coefficients can be defined as

$$\begin{align*}
a_n' &= \frac{1}{\pi} \int_{-\pi}^{\pi} A'_i(\phi) \cos(n\phi) d\phi \\
b_n' &= \frac{1}{\pi} \int_{-\pi}^{\pi} A'_i(\phi) \sin(n\phi) d\phi.
\end{align*}$$  \hspace{1cm} (8)

In \cite{10}, the author computes the phase error $\phi$ overall a data block; in this paper, we compute the phase error $\phi$ by a single symbol, ignoring the constant and higher harmonic terms, thus (7) can be rewritten as

$$A'_{BPSK}(\phi) = a_0' \cos(\phi - \phi_i) + a_1' \cos(2\phi - 2\phi_i) + a_2' \cos(2\phi - 2\phi_i).$$  \hspace{1cm} (9)

where $a_0$, $a_1$, and $a_2$ are the final magnitude of two harmonics, and $\theta_0$ and $\theta_2$ are the final arguments, which are obtained by $i$th symbol, such as

$$\begin{align*}
a_0 &= \cos(\theta_i) = a_0' \cos(\phi_i) \\
a_1 &= \sin(\theta_i) = a_0' \sin(\phi_i) \\
a_2 &= \cos(\theta_i) = a_0' \cos(2\phi_i) \\
a_2 &= \sin(\theta_i) = a_0' \sin(2\phi_i).
\end{align*}$$  \hspace{1cm} (10)

In order to get the ML estimation, we need to derive (9) and set the derivative of (9) equal to zero. So we get

$$\frac{dA'_{BPSK}}{d\phi} = a_0 \sin(\phi - \theta_i) + 2a_0 \sin(2\phi - \theta_i) = 0 .$$  \hspace{1cm} (11)

Considering $\sin x \approx x$ when $x$ is a small value, and then we get the ML estimation of $\phi$. Let $\hat{\phi}_{\text{ML}}$ denote the
ML estimation value of $\phi$, it can be easily attained by

$$\hat{\phi}_{\text{ML}} = \frac{\alpha_1 \beta_1 + 2 \alpha_2 \beta_3}{\alpha_1 + 4 \alpha_2}$$ \quad (12)

Attention is required in the simulation, in order to trace a small frequency offset, we introduce a loop filter in the loop, and the transfer function of the loop filter can be represented by $H(z)$

$$H(z) = \frac{K_p + K_i z^{-1}}{1 - z^{-1}}$$ \quad (13)

where $K_p$ and $K_i$ are the gains of the filter.

### 4. Tracking the Large Frequency Offset

A PLL-based circuit can be effective for tracking small frequency offset, but it is generally insufficient to track large frequency offset. In this paper, we adopt a simplified frequency search method called 2 step frequency search. For the first search we set a larger sweep step, in which the frequency offset equals or approaches to the real frequency offset, but it is generally insufficient to track small frequency offset, we introduce a loop filter in the loop. The frequency search method called 2 step frequency search.

For BPSK

$$E_b[a^* | r, b^{(n)}] = \tanh \frac{L(i)}{2}$$

where $L(i)$ is the extrinsic information obtained from the Turbo decoder.

### 5. Simulation Results

In this simulation, all the parameters are given in section 2. The curve of Fig. 2 appears as a straight line with gradient very close to 1 between $|\phi| \leq 80^\circ$, which means that the phase estimator produces an accurate estimation in this range. However, out of this range the estimation will be inaccurate.

Fig. 3 shows the relationship between the objective function and frequency offset. When the candidate frequency offset equals or approaches to the real frequency offset, the value of the objective function will be the maximization. In Fig. 3, three curves represent the objective functions with different step size of 100 ppm, 200 ppm, and 600 ppm, respectively. If the step is much larger, such as 600 ppm, the search will fail.
The BER performance for BPSK with frequency offset in the range of \([-150 \text{ ppm}, 150 \text{ ppm}\)] is depicted in Fig. 4. It shows that this technique ensures that the decoding is robust with a remarkably wide range of frequency offset.

Fig. 5 shows the BER performance of the improved APPA algorithm. We can see that the performance of the improved APPA is very close to the ideal synchronized situation when the phase error is less than 80° and the frequency offset is less than 100 ppm.

Fig. 6 depicts the BER performance of the receiver shown in the Fig. 1. The simulation results show that the algorithm can work successfully with large frequency offset and phase offset when SNR is larger than \(-7.4 \text{ dB}\) and its performance approaches to the optimally synchronized system.

### 6. Conclusions

In [10], Zhang and Burr proposed a joint decoding and phase recovery method called APPA which estimated the phase error based on the whole data block. Therefore, this method is suitable only for phase estimation with zero frequency offset. In this paper, an improved residual carrier frequency offset estimation algorithm based on APPA phase estimation is proposed. It computes the phase error symbol by symbol, while, to trace large frequency, a frequency-sweep module is proposed. The simulation results show that the algorithm can work successfully with small frequency offset and phase error at very low SNR and its performance is very close to the optimally synchronized system.

### References


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